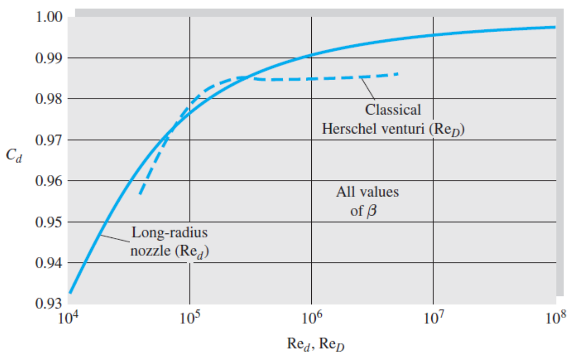
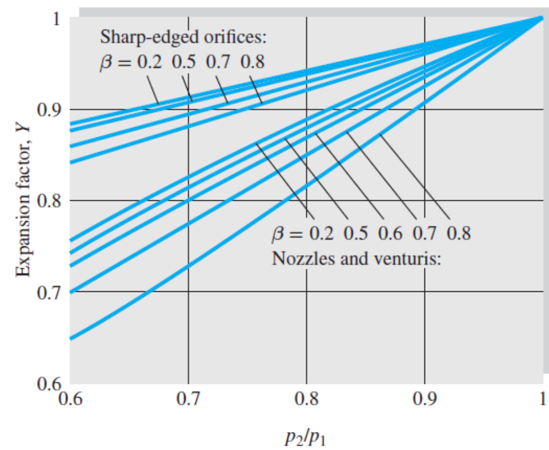
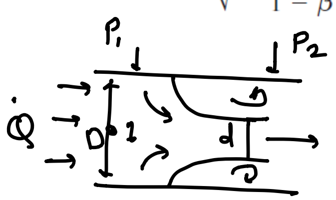
A long-radius nozzle of diameter 6 cm is used to meter airflow in a 10-cm-diameter pipe. Upstream conditions are $p_1 = 200$ kPa and $T_1 = 100$°C. If the pressure drop through the nozzle is 60 kPa, estimate the flow rate in m^3/s.





$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - \beta^4}} \quad \text{where} \quad \beta = \frac{d}{D}$$



$$\beta = \frac{d}{D}$$

$$\dot{Q} = C_d A_t \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Incompressible fluid $\frac{P_2}{P_1} \sim 1$ $P_1 = 100$ kPa, $P_2 = 99$ kPa

Compressible fluid $\frac{P_2}{P_1} < 1 \rightarrow$ $$Q = C_d Y A_t \sqrt{\frac{2(P_1 - P_2)}{\rho_1(1-\beta^4)}}$$

$P_2 - P_1 = 60$ kPa $\rightarrow$ $P_2 = 140$ kPa. $P = \rho R T$

200 kPa $\sim$ 30% $\rightarrow$ Compressible flow.

$$\frac{P_2}{P_1} = \frac{140}{200} = 0.7$$

$$\beta = \frac{d}{D} = \frac{6\text{ cm}}{10\text{ cm}} = 0.6$$

$$\rho_1 = \frac{P_1}{R T_1} = \frac{200000}{287(100+273)} \approx 1.87 \text{ kg/m}^3$$

@ $T = 100$°C, $\mu_{air} = 2.17 \times 10^{-5}$ kg/m s

From Y vs P_2/P_1 above $\rightarrow$ $Y = 0.8$ for $P_2/P_1 = 0.7$ & $\beta = 0.6$

$$\dot{m} = \rho_1 \dot{Q} = C_d Y A_t \sqrt{\frac{2\rho_1 (P_1 - P_2)}{1-\beta^4}}$$

↑ Guess $C_d \approx 0.98$ & $Re_D \approx 10^5$

$$\dot{m} = 0.98 \times 0.8 \times \frac{\pi}{4}(0.06)^2 \sqrt{\frac{2 \times 1.87\ kg/m^3 (60 \times 10^3)\ Pa}{1-(0.6)^4}}$$

$$\dot{m} \approx 1.13\ kg/s. \rightarrow V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \frac{\pi}{4} d^2}$$

$$Re_d = \frac{\rho V d}{\mu} = \frac{4\dot{m}}{\pi \mu d} = \frac{4 \times 1.13}{\pi \times 2.17 \times 10^{-5} \times 0.06}$$

$$Re_d \approx 1.11 \times 10^6$$

Correct C_d & Re_d

Guess $Re_d = 10^6$ ~ $C_d = 0.99$,

Repeat $\dot{m}$ calculation

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1 (P_1 - P_2)}{(1-\beta^4)}} \approx 1.14\ kg/s$$

(C_d = 0.99)

$$Re_d = \frac{4\dot{m}}{\pi \mu d} \approx 10^6$$

$\dot{m} = 1.14\ kg/s$ – $Re_d = 10^6$