In the late 1940s, much of the science concerning nuclear bombs was highly classified. In particular, information regarding the energy released in a nuclear explosion, *e.g.* the number of equivalent kilotons of TNT (nowadays the energy is measured in megatons), was top secret. G.I. Taylor, a famous fluid mechanics professor, was asked in 1941 by the British Civil Defence Research Committee of the Ministry of Home Security to predict the dynamics of a blast caused by a nuclear explosion. In his analysis, Taylor assumed that a finite amount of energy, E, is suddenly released in an infinitely concentrated form. The resulting blast wave, with a radius R, then propagates into the surrounding atmosphere, with density ρ_0 and specific heat ratio $\gamma = c_p/c_v$, as a function of time, t. Taylor's analysis resulted in a simple relationship between the blast radius as a function of the time, air density, blast energy, and specific heat ratio. Using declassified photographs of the first nuclear explosion, which occurred at the Trinity test site in New Mexico in 1945, Taylor was able to estimate the energy release to within remarkable accuracy.

Perform a dimensional analysis to determine an expression involving the blast radius as a function of the other significant parameters in the problem.

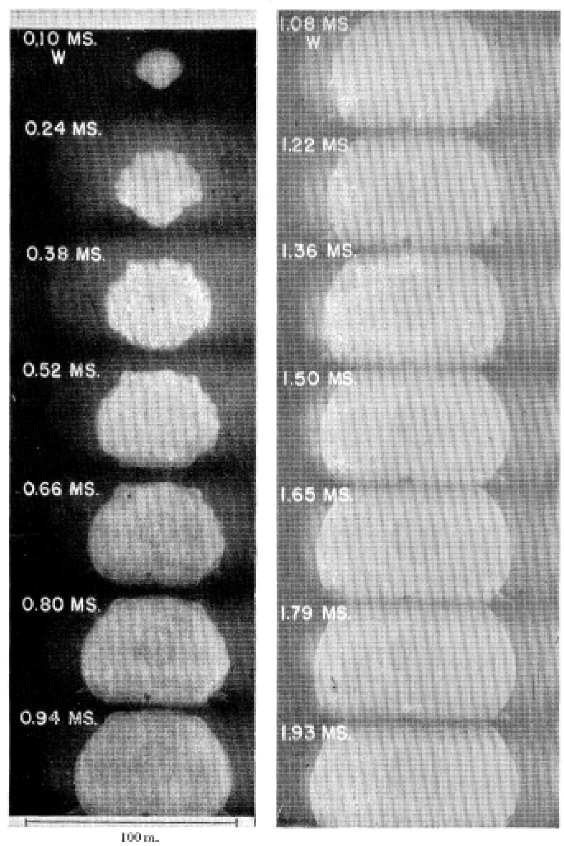

FIGURE 6. Succession of photographs of the 'ball of fire' from $t = 0{\cdot}10$ msec. to $1{\cdot}93$ msec.

References: Taylor, G., 1950, "The formation of a blast wave by a very intense explosion. I. Theoretical analyses," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 201, No. 1065, pp. 159 – 174. Taylor, G., 1950, "The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 201, No. 1065, pp. 175 – 186.

SOLUTION:

1. Write the dimensional functional relationship.

$$R = f_1\left(t, E, \rho_0, \gamma\right) \quad (1)$$

2. Determine the basic dimensions of each parameter.

$[R] = L$
$[t] = T$
$[E] = FL = ML^2/T^2$
$[\rho_0] = M/L^3$
$[\gamma] = -$

3. Determine the number of Π terms required to describe the functional relationship.

\# of variables = 5
\# of reference dimensions = 3 (L, T, M)

(# Π terms) = (# of variables) – (# of reference dimensions) = 5 – 3 = 2

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions): t, E, ρ_0.

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = R t^a E^b \rho_0^c \quad (2)$$

$$\Rightarrow \ M^0 L^0 T^0 = \left(\frac{L}{1}\right)\left(\frac{T}{1}\right)^a \left(\frac{ML^2}{T^2}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{array}{llll} M: & 0 = b + c & & a = -\frac{2}{5} \\ L: & 0 = 1 + 2b - 3c & \Rightarrow & b = -\frac{1}{5} \\ T: & 0 = a - 2b & & c = \frac{1}{5} \end{array}$$

$$\therefore \Pi_1 = \frac{R\rho_0^{1/5}}{t^{2/5} E^{1/5}} \quad (3)$$

$$\Pi_2 = \gamma \quad \text{(the specific heat ratio is already a dimensionless quantity)} \quad (4)$$

6. Verify that each Π term is, in fact, dimensionless.

$$\left[\Pi_1\right] = \left[\frac{R\rho_0^{1/5}}{t^{2/5} E^{1/5}}\right] = \frac{L}{1}\frac{M^{1/5}}{L^{3/5}}\frac{1}{T^{2/5}}\frac{T^{2/5}}{M^{1/5} L^{2/5}} = 1 \ \text{ OK!} \quad (5)$$

$$\left[\Pi_2\right] = \left[\gamma\right] = - \ \text{ OK!} \quad (6)$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{R\rho_0^{1/5}}{t^{2/5} E^{1/5}} = f_2\left(\gamma\right)} \quad (7)$$

Note that the specific heat ratio and density of the atmosphere are well known (and assumed constant) so we could write Eq. (7) as:

$$R = c_1 t^{2/5} E^{1/5} \tag{8}$$

where c_1 is a constant (involving ρ_0 and γ). Taking the base 10 logarithm of both sides and re-arranging:

$$\tfrac{5}{2}\log_{10} R = \log_{10} t + \tfrac{5}{2}\left(\log_{10} c_1 + \tfrac{1}{5}\log_{10} E\right) \tag{9}$$

Thus, for a given explosion, the blast radius should follow a straight line when $(5/2)\log_{10}R$ is plotted as a function of $\log_{10}t$. The intercept of the line will be related to the atmospheric conditions (recall that c_1 is a function of ρ_0 and γ) and the blast energy, E. The following plots shows the measurements made from the video of the 1945 nuclear test. It's remarkable that the predictions performed by Taylor four years before the actual test are so accurate. In addition, simple radius vs. time measurements from a movie of the explosion could also easily give an estimate of the energy released. Taylor estimated the energy release to be between 23.7 kilotons of TNT. The actual energy release was estimated to be 20 kilotons.

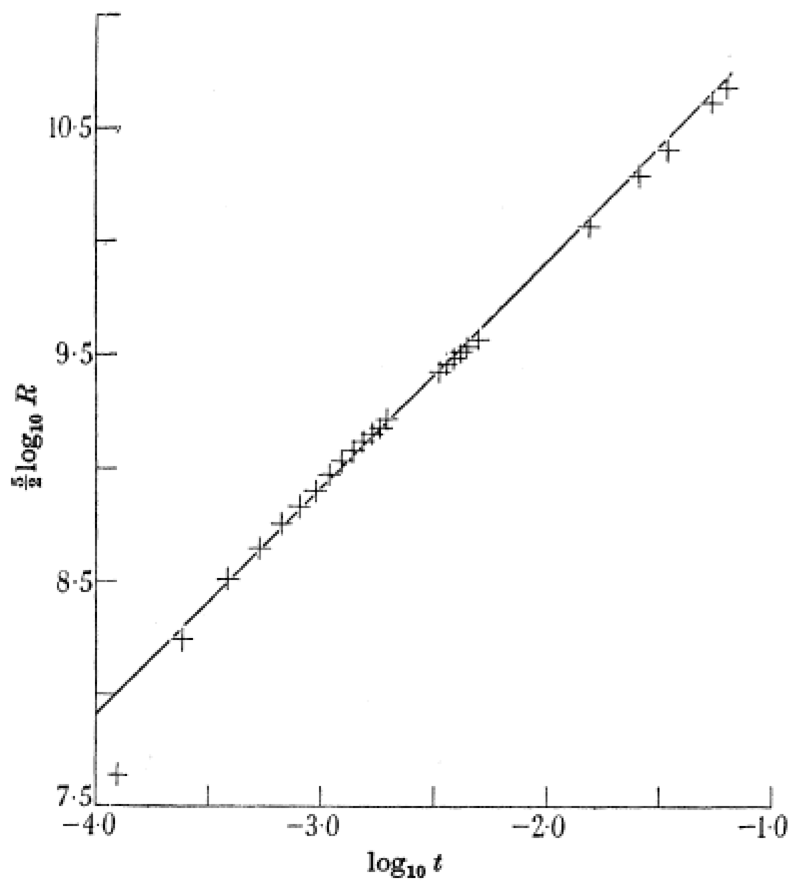

FIGURE 1. Logarithmic plot showing that $R^{5/2}$ is proportional to t.