Spin plays an important role in the flight trajectory of golf, Ping-Pong, and tennis balls. Therefore, it is important to know the rate at which spin decreases for a ball in flight. The aerodynamic torque, T, acting on a ball in flight, is thought to depend on flight speed, V, air density, ρ, air viscosity, μ, ball diameter, D, spin rate (angular speed), ω, and diameter of the dimples on the ball, d. Determine the dimensionless parameters that result.

SOLUTION:

1. Write the dimensional functional relationship.
   $$T = f_1\left(V, \rho, \mu, D, \omega, d\right)$$

2. Determine the basic dimensions of each parameter.
   $$[T] = F \cdot L = ML^2/T^2$$
   $$[V] = L/T$$
   $$[\rho] = M/L^3$$
   $$[\mu] = M/LT$$
   $$[D] = L$$
   $$[\omega] = 1/T$$
   $$[d] = L$$

3. Determine the number of Π terms required to describe the functional relationship.
   # of variables = 7 (T, V, ρ, μ, D, ω, d)
   # of reference dimensions = 3 (M, L, T)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 7 – 3 = 4

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).
   ρ, V, D (Note that these repeating variables have independent dimensions.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.
   $$\Pi_1 = T\rho^a V^b D^c$$
   $$\Rightarrow \; M^0 L^0 T^0 = \left(ML^2/T^2\right)\left(M/L^3\right)^a \left(L/T\right)^b \left(L/1\right)^c$$
   M: $0 = 1 + a \quad \Rightarrow \quad a = -1$
   T: $0 = -2 - b \quad \Rightarrow \quad b = -2$
   L: $0 = 2 - 3a + b + c \quad \Rightarrow \quad c = -3$
   $$\therefore \Pi_1 = \frac{T}{\rho V^2 D^3}$$

$$\Pi_2 = \mu\rho^a V^b D^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{M}{LT}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b \left(\frac{L}{1}\right)^c$$

M: $\quad 0 = 1 + a \quad \Rightarrow \; a = -1$

T: $\quad 0 = -1 - b \quad \Rightarrow \; b = -1$

L: $\quad 0 = -1 - 3a + b + c \quad \Rightarrow \; c = -1$

$$\therefore \Pi_2 = \frac{\mu}{\rho VD} \text{ or } \Pi_2 = \frac{\rho VD}{\mu} \quad \text{(a Reynolds number!)}$$

$$\Pi_3 = \omega\rho^a V^b D^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{1}{T}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b \left(\frac{L}{1}\right)^c$$

M: $\quad 0 = a \quad \Rightarrow \; a = 0$

T: $\quad 0 = -1 - b \quad \Rightarrow \; b = -1$

L: $\quad 0 = -3a + b + c \quad \Rightarrow \; c = 1$

$$\therefore \Pi_3 = \frac{\omega D}{V}$$

$$\Pi_4 = d\,\rho^a V^b D^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{L}{1}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b \left(\frac{L}{1}\right)^c$$

M: $\quad 0 = a \quad \Rightarrow \; a = 0$

T: $\quad 0 = -b \quad \Rightarrow \; b = 0$

L: $\quad 0 = 1 + c \quad \Rightarrow \; c = -1$

$$\therefore \Pi_4 = \frac{d}{D}$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{T}{\rho V^2 D^3}\right] = \frac{ML^2}{T^2}\frac{L^3}{M}\frac{T^2}{L^2}\frac{1}{L^3} = 1 \text{ OK!}$$

$$[\Pi_2] = \left[\frac{\rho VD}{\mu}\right] = \frac{M}{L^3}\frac{L}{T}\frac{L}{1}\frac{LT}{M} = 1 \text{ OK!}$$

$$[\Pi_3] = \left[\frac{\omega D}{V}\right] = \frac{1}{T}\frac{L}{1}\frac{T}{L} = 1 \text{ OK!}$$

$$[\Pi_4] = \left[\frac{d}{D}\right] = \frac{L}{1}\frac{1}{L} = 1 \text{ OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{T}{\rho V^2 D^3} = f_2\left(\underbrace{\frac{\rho VD}{\mu}}_{\text{Reynolds \#}}, \frac{\omega D}{V}, \frac{d}{D}\right)} \qquad (1)$$