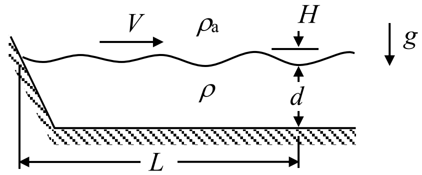
It is desired to determine the wave height when wind blows across a lake. The wave height, *H*, is assumed to be a function of the wind speed, *V*, the water density, ρ, the air density, ρ_a, the water depth, *d*, the distance from the shore, *L*, and the acceleration of gravity, *g*. Use *d*, *V*, and ρ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.

SOLUTION:



1. Write the dimensional functional relationship.
   $H = f_1(V, \rho, \rho_a, d, L, g)$

2. Determine the basic dimensions of each parameter.
   $[H] = L$
   $[V] = L/T$
   $[\rho] = M/L^3$
   $[\rho_a] = M/L^3$
   $[d] = L$
   $[L] = L$
   $[g] = L/T^2$

3. Determine the number of Π terms required to describe the functional relationship.
   \# of variables = 7 (H, V, ρ, ρ_a, d, L, g)
   \# of reference dimensions = 3 (L, T, M)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 7 – 3 = 4

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).
   d, V, ρ

5. Make the remaining non-repeating variables dimensionless using the repeating variables.
   $\Pi_1 = \dfrac{H}{d}$ (by inspection)

   $\Pi_2 = \dfrac{\rho_a}{\rho}$ (by inspection)

   $\Pi_3 = \dfrac{L}{d}$ (by inspection)

   $\Pi_4 = \dfrac{V}{\sqrt{gd}}$ (by inspection, This is a Froude number!)

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[{}^{H}\!/_{d} \right] = {}^{L}\!/_{1}\ {}^{1}\!/_{L} = 1 \quad \text{OK!}$$

$$[\Pi_2] = \left[{}^{\rho_a}\!/_{\rho} \right] = {}^{M}\!/_{L^3}\ {}^{L^3}\!/_{M} = 1 \quad \text{OK!}$$

$$[\Pi_3] = \left[{}^{L}\!/_{d} \right] = {}^{L}\!/_{1}\ {}^{1}\!/_{L} = 1 \quad \text{OK!}$$

$$[\Pi_4] = \left[{}^{V}\!/_{\sqrt{gd}} \right] = {}^{L}\!/_{T}\ {}^{T}\!/_{L^{1/2}}\ {}^{1}\!/_{L^{1/2}} = 1 \quad \text{OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{H}{d} = f_2\left(\frac{\rho_a}{\rho}, \frac{L}{d}, \frac{V}{\sqrt{gd}} \right)}$$