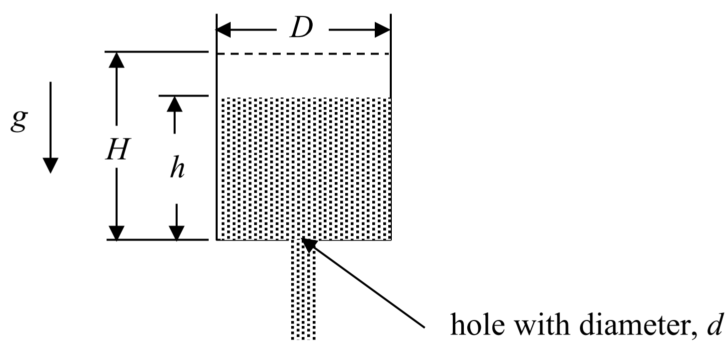
The height of the free surface, *h*, in a tank of diameter, *D*, that is draining fluid through a small hole at the bottom with diameter, *d*, decreases with time, *t*. This change in free surface height is studied experimentally with a half-scale model. For the prototype tank:

H = 16 in. (the initial height of the free surface)
D = 4.0 in.
d = 0.25 in.

Experimental data is obtained from the prototype and half-scale model and is given below:

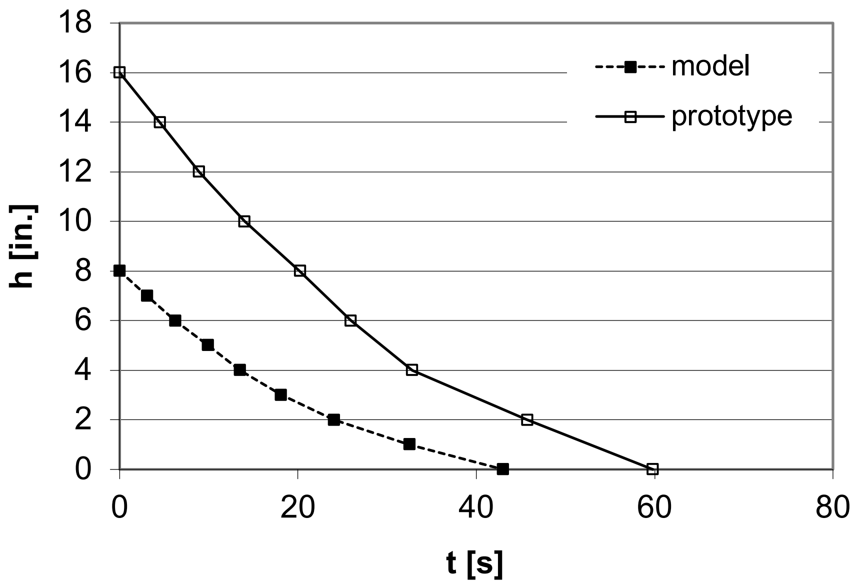
Model Data		Prototype Data	
h [in.]	*t* [s]	*h* [in.]	*t* [s]
8.0	0.0	16.0	0.0
7.0	3.1	14.0	4.5
6.0	6.2	12.0	8.9
5.0	9.9	10.0	14.0
4.0	13.5	8.0	20.2
3.0	18.1	6.0	25.9
2.0	24.0	4.0	32.8
1.0	32.5	2.0	45.7
0.0	43.0	0.0	59.8

1. Plot, on the same graph, the height data as a function of time for both the model and the prototype.
2. Develop a set of dimensionless parameters for this problem assuming that: $h = f(H, D, d, g, t)$
3. Re-plot, on the same graph, the height data as a function of time in non-dimensional form for both the model and prototype.



SOLUTION:

First plot the model and prototype dimensional data.



Now perform a dimensional analysis to determine the dimensionless terms describing the relationship.

1. Write the dimensional functional relationship.

$$h = f_1(H, D, d, g, t)$$

2. Determine the basic dimensions of each parameter.

$$[h] = L$$
$$[H] = L$$
$$[D] = L$$
$$[d] = L$$
$$[g] = L/T^2$$
$$[t] = T$$

3. Determine the number of Π terms required to describe the functional relationship.

   \# of variables = 6 (h, H, D, d, g, t)
   \# of reference dimensions = 2 (L, T)
   (Note that the number of reference dimensions and the number of basic dimensions are equal for this problem.)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 6 – 2 = 4

4. Choose two repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   H, g (Note that the dimensions for these variables are independent.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$\Pi_1 = \frac{h}{H}$ (Found via inspection.)

$\Pi_2 = \frac{D}{H}$ (Found via inspection.)

$\Pi_3 = \frac{d}{H}$ (Found via inspection.)

$\Pi_4 = tH^a g^b$

$$\Rightarrow \; L^0 T^0 = \left(\frac{T}{1}\right)\left(\frac{L}{1}\right)^a \left(\frac{L}{T^2}\right)^b$$

$$\begin{aligned} L:&\quad 0 = a + b \\ T:&\quad 0 = 1 - 2b \end{aligned} \;\Rightarrow\; \begin{aligned} a &= -\tfrac{1}{2} \\ b &= \tfrac{1}{2} \end{aligned}$$

$$\therefore \Pi_4 = t\sqrt{\frac{g}{H}}$$

6. Verify that each Π term is, in fact, dimensionless.

$[\Pi_1] = \left[\frac{h}{H}\right] = \frac{L}{1}\frac{1}{L} = 1$ OK!

$[\Pi_2] = \left[\frac{D}{H}\right] = \frac{L}{1}\frac{1}{L} = 1$ OK!

$[\Pi_3] = \left[\frac{d}{H}\right] = \frac{L}{1}\frac{1}{L} = 1$ OK!

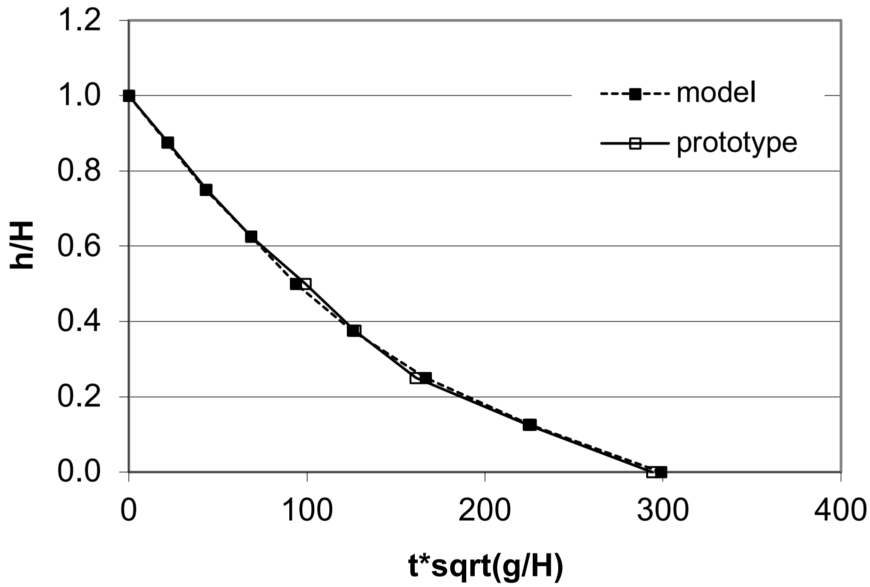
$[\Pi_4] = \left[t\sqrt{\frac{g}{H}}\right] = \frac{T}{1}\frac{L^{1/2}}{T}\frac{1}{L^{1/2}} = 1$ OK!

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{h}{H} = f_2\left(\frac{D}{H}, \frac{d}{H}, t\sqrt{\frac{g}{H}}\right)}$$

Now plot the model and prototype data in dimensionless form. Note that since there is geometric similarity (the model is one-half the size of the prototype):

$\left(\frac{d}{H}\right)_M = \left(\frac{d}{H}\right)_P$ and $\left(\frac{D}{H}\right)_M = \left(\frac{D}{H}\right)_P$



Notice that the data collapse to a single curve when plotted in dimensionless terms.