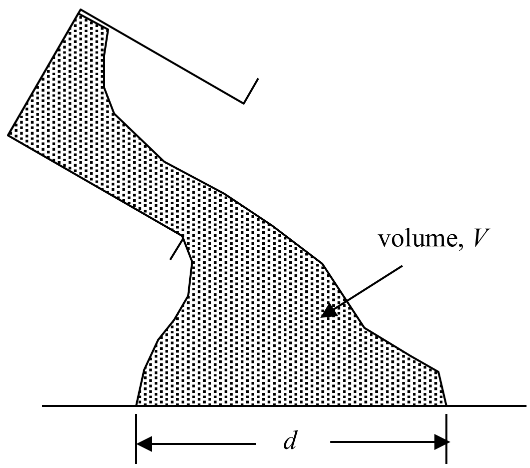
A viscous fluid is poured onto a horizontal plate as shown in the figure. Assume that the time, *t*, required for the fluid to flow a certain distance, *d*, along the plate is a function of the volume of fluid poured, *V*, acceleration due to gravity, *g*, fluid density, ρ, and fluid dynamic viscosity, μ. Determine an appropriate set of dimensionless terms to describe this process.



SOLUTION:

1. Write the dimensional functional relationship.

$$t = f_1(d, V, g, \rho, \mu)$$

2. Determine the basic dimensions of each parameter.

$$[t] = T$$
$$[d] = L$$
$$[V] = L^3$$
$$[g] = L/T^2$$
$$[\rho] = M/L^3$$
$$[\mu] = M/(LT)$$

3. Determine the number of Π terms required to describe the functional relationship.

   # of variables = 6 (t, d, V, g, ρ, μ)
   # of reference dimensions = 3 (T, L, M)
   (Note that the number of reference dimensions and the number of basic dimensions are equal for this problem.)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 6 – 3 = 3

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   d, g, ρ (Note that the dimensions for these variables are independent.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = t d^a g^b \rho^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{T}{1}\right)\left(\frac{L}{1}\right)^a \left(\frac{L}{T^2}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{aligned} M:&\quad 0 = c & & a = -\tfrac{1}{2} \\ L:&\quad 0 = a + b - 3c & \Rightarrow \; & b = \tfrac{1}{2} \\ T:&\quad 0 = 1 - 2b & & c = 0 \end{aligned}$$

$$\therefore \Pi_1 = t\sqrt{\frac{g}{d}}$$

$$\Pi_2 = V d^a g^b \rho^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{L^3}{1}\right)\left(\frac{L}{1}\right)^a \left(\frac{L}{T^2}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{aligned} M:&\quad 0 = c & & a = -3 \\ L:&\quad 0 = 3 + a + b - 3c & \Rightarrow \; & b = 0 \\ T:&\quad 0 = -2b & & c = 0 \end{aligned}$$

$$\therefore \Pi_2 = \frac{V}{d^3}$$

$$\Pi_3 = \mu d^a g^b \rho^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{M}{LT}\right)\left(\frac{L}{1}\right)^a \left(\frac{L}{T^2}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{aligned} M:&\quad 0 = 1 + c & & a = -\tfrac{3}{2} \\ L:&\quad 0 = -1 + a + b - 3c & \Rightarrow \; & b = -\tfrac{1}{2} \\ T:&\quad 0 = -1 - 2b & & c = -1 \end{aligned}$$

$$\therefore \Pi_3 = \frac{\mu}{\rho d\sqrt{gd}}$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[t\sqrt{\frac{g}{d}}\right] = \frac{T}{1}\frac{L^{1/2}}{T}\frac{1}{L^{1/2}} = 1 \quad \text{OK!}$$

$$[\Pi_2] = \left[\frac{V}{d^3}\right] = \frac{L^3}{1}\frac{1}{L^3} = 1 \quad \text{OK!}$$

$$[\Pi_3] = \left[\frac{\mu}{\rho d\sqrt{gd}}\right] = \frac{M}{LT}\frac{L^3}{M}\frac{1}{L}\frac{T}{L^{1/2}}\frac{1}{L^{1/2}} = 1 \quad \text{OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{t\sqrt{\frac{g}{d}} = f_2\left(\frac{V}{d^3}, \frac{\mu}{\rho d\sqrt{gd}}\right)}$$