An open cylindrical tank having a diameter *D* is supported around its bottom circumference and is filled to a depth *h* with a liquid having a specific weight γ. The vertical deflection, δ, of the center of the bottom is a function of *D*, *h*, *d*, γ, and *E* where *d* is the thickness of the bottom and *E* is the modulus of elasticity of the bottom material. Form the dimensionless groups describing this relationship.

SOLUTION:

1. Write the dimensional functional relationship.

$$\delta = f_1\left(D, h, d, \gamma, E\right)$$

2. Determine the basic dimensions of each parameter.

$$[\delta] = L$$
$$[h] = L$$
$$[D] = L$$
$$[d] = L$$
$$[\gamma] = \frac{M}{L^2T^2} = \frac{F}{L^3}$$
$$[E] = \frac{M}{LT^2} = \frac{F}{L^2}$$

3. Determine the number of Π terms required to describe the functional relationship.

   # of variables = 6 (δ, D, h, d, γ, E)
   # of reference dimensions = 2 (L, F/L^2 or L, M/T^2)
   (Note that the number of reference dimensions and the number of basic dimensions are not the same for this problem!)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 6 – 2 = 4

4. Choose two repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   D, γ (Note that the dimensions for D and γ are independent.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = \delta D^a \gamma^b$$

$$\Rightarrow \; F^0 L^0 = \left(L/1\right)\left(L/1\right)^a \left(F/L^3\right)^b$$

$$F: \; 0 = b$$

$$L: \; 0 = 1 + a - 3b \; \Rightarrow \; a = -1$$

$$\therefore \Pi_1 = \delta/D$$

$$\Pi_2 = h D^a \gamma^b$$

$$\Rightarrow \; F^0 L^0 = \left(L/1\right)\left(L/1\right)^a \left(F/L^3\right)^b$$

$$F: \; 0 = b$$

$$L: \; 0 = 1 + a - 3b \; \Rightarrow \; a = -1$$

$$\therefore \Pi_2 = h/D$$

$$\Pi_3 = d D^a \gamma^b$$

$$\Rightarrow \; F^0 L^0 = \left(L/1\right)\left(L/1\right)^a \left(F/L^3\right)^b$$

$$F: \; 0 = b$$

$$L: \; 0 = 1 + a - 3b \; \Rightarrow \; a = -1$$

$$\therefore \Pi_3 = d/D$$

$$\Pi_4 = E D^a \gamma^b$$

$$\Rightarrow \; F^0 L^0 = \left(F/L^2\right)\left(L/1\right)^a \left(F/L^3\right)^b$$

$$F: \; 0 = 1 + b \; \Rightarrow \; b = -1$$

$$L: \; 0 = -2 + a - 3b \; \Rightarrow \; a = -1$$

$$\therefore \Pi_4 = E/(D\gamma)$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\delta/D\right] = L/L = 1 \; \text{ OK!}$$

$$[\Pi_2] = \left[h/D\right] = L/L = 1 \; \text{ OK!}$$

$$[\Pi_3] = \left[d/D\right] = L/L = 1 \; \text{ OK!}$$

$$[\Pi_4] = \left[E/(D\gamma)\right] = \left(F/L^2\right)\left(1/L\right)\left(L^3/F\right) = 1 \; \text{ OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{\delta}{D} = f_2\left(\frac{h}{D}, \frac{d}{D}, \frac{E}{D\gamma}\right)}$$