The power, P, to drive an axial flow pump depends on the following variables:

density of the fluid, ρ
angular speed of the rotor, Ω
diameter of the rotor, D
head rise across the pump, ΔH ($= \Delta p/\rho g$)
volumetric flow through the pump, Q

a. Rewrite the functional relationship in dimensionless form.
b. A model scaled to one-third the size of the prototype has the following characteristics:

$\Omega_m = 900$ rpm
$D_m = 5$ in
$\Delta H_m = 10$ ft
$Q_m = 3$ ft^3/s
$P_m = 2$ hp

If the full-size pump is to run at 300 rpm, what is the power required for this pump? What head will the pump maintain? What will the volumetric flow rate be in the prototype?

SOLUTION:

1. Write the dimensional functional relationship.

   $$P = f_1\left(\rho, \Omega, D, \Delta H, Q\right)$$

2. Determine the basic dimensions of each parameter.

   $$\left[P\right] = FL/T = ML^2/T^3$$
   $$\left[\rho\right] = M/L^3$$
   $$\left[\Omega\right] = 1/T$$
   $$\left[D\right] = L$$
   $$\left[\Delta H\right] = L$$
   $$\left[Q\right] = L^3/T$$

3. Determine the number of Π terms required to describe the functional relationship.

   \# of variables = 6 (P, ρ, Ω, D, ΔH, Q)
   \# of reference dimensions = 3 (M, L, T)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 6 – 3 = 3

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   ρ, D, Ω (Note that these repeating variables have independent dimensions.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = P\rho^a D^b \Omega^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{ML^2}{T^3}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{1}\right)^b \left(\frac{1}{T}\right)^c$$

$$\begin{aligned} M&: \quad 0 = 1 + a && \Rightarrow \; a = -1 \\ L&: \quad 0 = 2 - 3a + b && \Rightarrow \; b = -5 \\ T&: \quad 0 = -3 - c && \Rightarrow \; c = -3 \end{aligned}$$

$$\therefore \Pi_1 = \frac{P}{\rho \Omega^3 D^5}$$

$$\Pi_2 = \Delta H \rho^a D^b \Omega^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{L}{1}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{1}\right)^b \left(\frac{1}{T}\right)^c$$

$$\begin{aligned} M&: \quad 0 = a && \Rightarrow \; a = 0 \\ L&: \quad 0 = 1 - 3a + b && \Rightarrow \; b = -1 \\ T&: \quad 0 = -c && \Rightarrow \; c = 0 \end{aligned}$$

$$\therefore \Pi_2 = \frac{\Delta H}{D}$$

$$\Pi_3 = Q\rho^a D^b \Omega^c$$

$$M^0 L^0 T^0 = \left(\frac{L^3}{T}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{1}\right)^b \left(\frac{1}{T}\right)^c$$

$$\begin{aligned} M&: \quad 0 = a && \Rightarrow \; a = 0 \\ L&: \quad 0 = 3 - 3a + b && \Rightarrow \; b = -3 \\ T&: \quad 0 = -1 - c && \Rightarrow \; c = -1 \end{aligned}$$

$$\therefore \Pi_3 = \frac{Q}{\Omega D^3}$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{P}{\rho \Omega^3 D^5}\right] = \frac{ML^2}{T^3}\frac{L^3}{M}\frac{T^3}{1}\frac{1}{L^5} = 1 \quad \text{OK!}$$

$$[\Pi_2] = \left[\frac{\Delta H}{D}\right] = \frac{L}{1}\frac{1}{L} = 1 \quad \text{OK!}$$

$$[\Pi_3] = \left[\frac{Q}{\Omega D^3}\right] = \frac{L^3}{T}\frac{T}{1}\frac{1}{L^3} = 1 \quad \text{OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{P}{\rho \Omega^3 D^5} = f_2\left(\frac{\Delta H}{D}, \frac{Q}{\Omega D^3}\right)}$$

Now perform a scaling analysis assuming that the same fluid is used in the model and prototype, i.e., $\rho_M = \rho_P$. Note that since a one-third scale model is being used, $D_P/D_M = 3/1$.

$$\left(\frac{P}{\rho\Omega^3 D^5}\right)_M = \left(\frac{P}{\rho\Omega^3 D^5}\right)_P$$

$$P_P = P_M\left(\frac{\Omega_P}{\Omega_M}\right)^3\left(\frac{D_P}{D_M}\right)^5$$

$$P_P = (2\text{ hp})\left(\frac{300\text{ rpm}}{900\text{ rpm}}\right)^3\left(\frac{3}{1}\right)^5$$

$$\boxed{P_P = 18\text{ hp}}$$

$$\left(\frac{\Delta H}{D}\right)_M = \left(\frac{\Delta H}{D}\right)_P$$

$$\Delta H_P = \Delta H_M\left(\frac{D_P}{D_M}\right)$$

$$\Delta H_P = (10\text{ ft})\left(\frac{3}{1}\right)$$

$$\boxed{\Delta H_P = 30\text{ ft}}$$

$$\left(\frac{Q}{\Omega D^3}\right)_M = \left(\frac{Q}{\Omega D^3}\right)_P$$

$$Q_P = Q_M\left(\frac{\Omega_P}{\Omega_M}\right)\left(\frac{D_P}{D_M}\right)^3$$

$$\boxed{Q_P = 27\text{ ft}^3\text{/s}}$$