The $y$-velocity component of a steady, 2D, incompressible flow is given by: $u_{y}=3 x y-x^{2} y$
Determine the most general velocity component in the $x$-direction for this flow.

## SOLUTION:

Consider the continuity equation:

$$
\begin{aligned}
& \frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}=0 \\
& \frac{\partial u_{x}}{\partial x}=-\frac{\partial u_{y}}{\partial y}=-\frac{\partial}{\partial y}\left(3 x y-x^{2} y\right)=-3 x+x^{2}
\end{aligned}
$$

Integrate $u_{x}$ with respect to $x$.

$$
\begin{equation*}
u_{x}=-\frac{3}{2} x^{2}+\frac{1}{3} x^{3}+f(y) \tag{2}
\end{equation*}
$$

where $f(y)$ is an unknown function of $y$.

