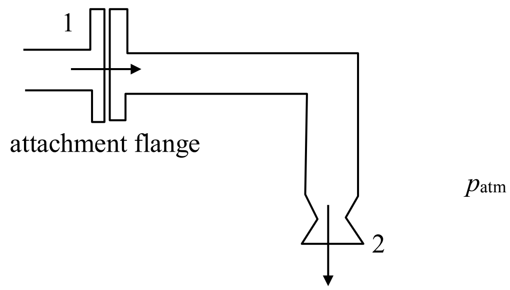
A steady flow of air passes through the elbow-nozzle assembly shown. At the inlet (1), the pipe diameter is $D_1 = 0.1524$ m and the air properties are $p_1 = 871.7$ kPa (abs), $T_1 = 529.0$ K, and $V_1 = 230.4$ m/s. The air is expanded through a converging-diverging nozzle discharging into the atmosphere where $p_{atm} = 101.3$ kPa (abs). At the nozzle exit (2), the nozzle diameter is $D_2 = 0.3221$ m and the air properties are $T_2 = 475.7$ K and $V_2 = 400.0$ m/s.



a. Is the flow through the elbow-nozzle assembly adiabatic?
b. Determine the components of the force in the attachment flange required to hold the elbow-nozzle assembly in place. You may neglect the effects of gravity.

SOLUTION:

If the flow is adiabatic in going from 1 to 2, then the energy equation will give:

$$T_1 + \frac{V_1^2}{2c_P} = T_2 + \frac{V_2^2}{2c_P} \qquad (1)$$

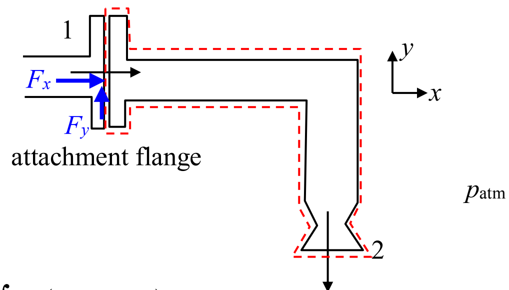
Using the given data:

T_1 = 529.0 K
V_1 = 230.4 m/s
c_P = 1004 J/(kg·K)
T_2 = 475.7 K
V_2 = 400.0 m/s

$$\Rightarrow \quad T_1 + \frac{V_1^2}{2c_P} = 555.4 \text{ K and } T_2 + \frac{V_2^2}{2c_P} = 555.1 \text{ K}$$

Since the stagnation temperatures are approximately the same, the flow can be considered adiabatic in going from 1 to 2.

To determine the force components, apply the linear momentum equation to the control volume shown below using the indicated fixed frame of reference.



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x \left(\rho \mathbf{u}_{rel} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x}$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{CS} u_x \left(\rho \mathbf{u}_{rel} \cdot d\mathbf{A}\right) = (V_1)\left(-\rho_1 V_1 \frac{\pi D_1^2}{4}\right) = -\rho_1 V_1^2 \frac{\pi D_1^2}{4}$$

$$F_{B,x} = 0$$

$$F_{S,x} = \left(p_1 - p_{atm}\right)\frac{\pi D_1^2}{4} + F_x$$

Substitute and simplify.

$$-\rho_1 V_1^2 \frac{\pi D_1^2}{4} = \left(p_1 - p_{atm}\right)\frac{\pi D_1^2}{4} + F_x$$

$$\boxed{F_x = -\rho_1 V_1^2 \frac{\pi D_1^2}{4} - \left(p_1 - p_{atm}\right)\frac{\pi D_1^2}{4}} \qquad (2)$$

Using the given numerical data:

$\rho_1 = p_1/(RT_1) = (871.7\text{ kPa})/[287\text{ J/(kg·K)}\cdot 529\text{ K}] = 5.738\text{ kg/m}^3$
$V_1 = 230.4\text{ m/s}$
$D_1 = 0.1524\text{ m}$
$p_1 = 871.7\text{ kPa (abs)}$
$p_{\text{atm}} = 101.3\text{ kPa (abs)}$
$\Rightarrow \boxed{F_x = -19.60\text{ kN}}$

Also note that:

$$\dot{m}_1 = \rho_1 V_1 \frac{\pi D_1^2}{4} = 24.11\text{ kg/s} \tag{3}$$

Now consider the *y*-direction.

$$\frac{d}{dt}\int_{\text{CV}} u_y \rho dV + \int_{\text{CS}} u_y \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,y} + F_{S,y}$$

where

$$\frac{d}{dt}\int_{\text{CV}} u_y \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{\text{CS}} u_y \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \left(-V_2\right)\left(\rho_2 V_2 \frac{\pi D_2^2}{4}\right) = -\rho_2 V_2^2 \frac{\pi D_2^2}{4}$$

$$F_{B,y} = 0$$

$$F_{S,y} = \left(p_2 - p_{\text{atm}}\right)\frac{\pi D_2^2}{4} + F_y$$

Substitute and simplify.

$$-\rho_2 V_2^2 \frac{\pi D_2^2}{4} = \left(p_2 - p_{\text{atm}}\right)\frac{\pi D_2^2}{4} + F_y$$

$$\boxed{F_y = -\rho_2 V_2^2 \frac{\pi D_2^2}{4} - \left(p_2 - p_{\text{atm}}\right)\frac{\pi D_2^2}{4}} \tag{4}$$

Note that from conservation of mass on the same control volume:

$$\dot{m}_1 = \dot{m}_2 = \rho_2 V_2 \frac{\pi D_2^2}{4} \tag{5}$$

The pressure at point 2 will depend on whether the flow at that point is subsonic or supersonic.

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{V_2}{\sqrt{\gamma R T_2}} \tag{6}$$

Using the given data:

$V_2 = 400.0\text{ m/s}$
$\gamma = 1.4$
$R = 287\text{ J/(kg·K)}$
$T_2 = 475.7\text{ K}$
$\Rightarrow c_2 = 437.2\text{ m/s} \Rightarrow \text{Ma}_2 = 0.9148 \Rightarrow$ <u>The flow at point 2 is subsonic.</u> $\Rightarrow$ <u>The pressure at point 2 is equal to atmospheric pressure, i.e., $p_2 = 101.3$ kPa (abs).</u>

Substituting the given numerical data in Eq. (4) gives $\boxed{F_y = -9.644\text{ kN}}$.