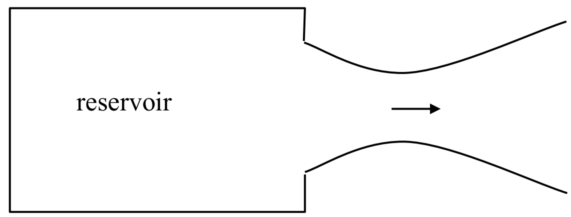
A rocket engine can be modeled as a reservoir of gas at high temperature feeding gas to a convergent/divergent nozzle as shown in the figure below.



For the questions below, assume the following:

1. The temperature in the reservoir is 3000 K.
2. The exhaust gases have the same properties as air: γ=1.4, R=287 J/(kg·K).
3. The exit Mach number is 2.5.
4. The rocket operates at design conditions (no shock waves or expansion waves present) where the surrounding pressure is $1*10^5$ Pa.
5. The area of the exit is $1*10^{-4}$ m^2.

Determine:

a. the temperature of the flow at the exit,
b. the pressure in the reservoir,
c. the throat area,
d. the mass flow rate out of the rocket,
e. the thrust produced by the rocket, and
f. sketch the process on a T-s diagram.

SOLUTION:

First determine the exit temperature using the adiabatic flow relation for stagnation temperature:

$$\frac{T_E}{T_0} = \left(1 + \tfrac{\gamma-1}{2}\mathrm{Ma}_E^2\right)^{-1} \Rightarrow \boxed{T_E = 1333\ \mathrm{K}} \tag{1}$$

using $T_0 = 3000$ K, $\gamma = 1.4$, and $\mathrm{Ma}_E = 2.5$.

Now determine the pressure in the reservoir using the isentropic stagnation pressure relation:

$$\frac{p_E}{p_0} = \left(1 + \tfrac{\gamma-1}{2}\mathrm{Ma}_E^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \boxed{p_0 = 1.709*10^6\ \mathrm{Pa}} \tag{2}$$

where $p_E = p_B = 1*10^5$ Pa (since the nozzle operates at design conditions, the exit pressure is equal to the back pressure), $\gamma = 1.4$, and $\mathrm{Ma}_E = 2.5$.

The throat area may be found using the isentropic sonic area ratio:

$$\frac{A_E}{A^*} = \frac{1}{\mathrm{Ma}_E}\left(\frac{1+\frac{\gamma-1}{2}\mathrm{Ma}_E^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow A^* = \boxed{A_T = 3.79*10^{-5}\ \mathrm{m}^2} \tag{3}$$

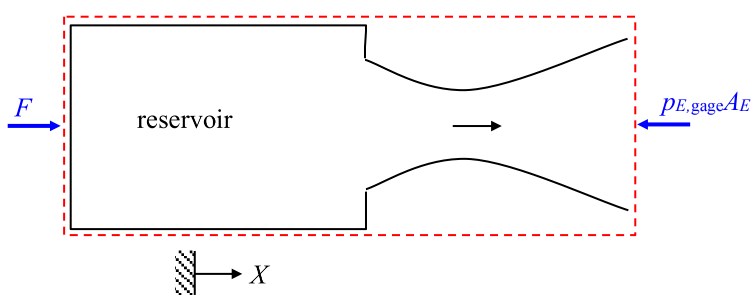
where $A_E = 1*10^{-4}$ m^2, $\gamma = 1.4$, and $\mathrm{Ma}_E = 2.5$. Note that since the flow starts from stagnation conditions and is supersonic at the exit, the throat area must also be the sonic area.

The mass flow rate may be found by considering the conditions at the exit:

$$\dot{m} = \rho_E V_E A_E = \left(\frac{p_E}{RT_E}\right)\left(\mathrm{Ma}_E\sqrt{\gamma R T_E}\right)A_E \Rightarrow \boxed{\dot{m} = 4.776*10^{-2}\ \mathrm{kg/s}} \tag{4}$$

where $p_E = p_B = 1*10^5$ Pa, $R = 287$ J/(kg·K), $T_E = 1333$ K, $\mathrm{Ma}_E = 2.5$, $\gamma = 1.4$, and $A_E = 1.0*10^{-4}$ m^2.

The thrust on the rocket may be found by applying the linear momentum equation in the x-direction on the control volume shown below.



$$\frac{d}{dt}\int_{\mathrm{CV}} u_X \rho dV + \int_{\mathrm{CS}} u_X\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = F_{B,X} + F_{S,X} \tag{5}$$

where

$$\frac{d}{dt}\int_{\mathrm{CV}} u_X \rho dV \approx 0 \quad \text{(most of the rocket mass inside the CV remains stationary)} \tag{6}$$

$$\int_{\mathrm{CS}} u_X\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{m}V_E = \dot{m}\left(\mathrm{Ma}_E\sqrt{\gamma R T_E}\right) \tag{7}$$

$$F_{B,X} = 0 \tag{8}$$

$$F_{S,X} = F - p_{E,\mathrm{gage}}A_E = F - \left(p_E - p_{\mathrm{atm}}\right)A_E \tag{9}$$

However, since the rocket is operating at design conditions, $p_E = p_B = p_{\mathrm{atm}}$.

Substitute and simplify.

$$F = \dot{m}V_E = \dot{m}\left(\mathrm{Ma}_E\sqrt{\gamma R T_E}\right) \;\Rightarrow\; \boxed{F = 87.4\ \mathrm{N}} \tag{10}$$

where $\dot{m} = 4.776 * 10^{-2}$ kg/s, $\mathrm{Ma}_E = 2.5$, $\gamma = 1.4$, $R = 287$ J/(kg·K), and $T_E = 1333$ K.