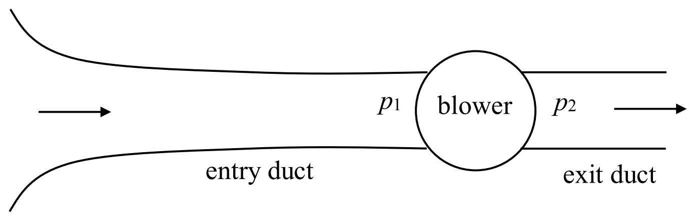
An air blower takes air from the atmosphere (100 kPa and 293 K) and ingests it through a smooth entry duct so that the losses are negligible. The cross-sectional area of the entry duct just upstream of the blower and that of the exit duct are both 0.01 m^2.

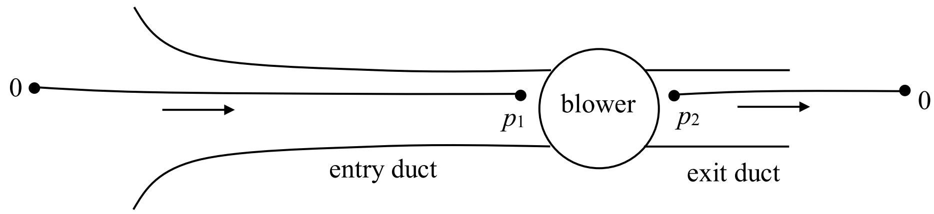


The pressure ratio, p_2/p_1, across the blower is 1.05 and the exit pressure is equal to atmospheric pressure. The air is assumed to behave isentropically upstream of the blower. Find:

a. the velocity of the air entering the blower, and
b. the mass flow rate of air through the system.

SOLUTION:

Apply conservation of energy between points 0 and 1 (refer to the figure below). Assume 1D, steady, isentropic (⇒ adiabatic) flow. Also neglect potential energy changes since a gas is the working fluid.



$$\dot{m}_1\left(h+\tfrac{1}{2}V^2\right)_1-\dot{m}_0\left(h+\tfrac{1}{2}V^2\right)_0=0 \qquad (1)$$

The velocity far upstream is negligible ($V_0 \approx 0$) and $\dot{m}_1=\dot{m}_0$. Substitute and simplify.

$$h_0=h_1+\tfrac{1}{2}V_1^2 \qquad (2)$$

Assume that the air behaves as a perfect gas ($\Delta h = c_P\Delta T$) and solve for V_1.

$$V_1=\sqrt{2c_P\left(T_0-T_1\right)} \qquad (3)$$

Express the temperature at point 1 in terms of a pressure ratio making use of the fact that the flow is isentropic.

$$\frac{T_1}{T_0}=\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \qquad (4)$$

The pressure rise across the blower is specified in the problem statement. Furthermore, the pressure at point 2 is equal to atmospheric pressure, i.e., $p_2 = p_0$. Re-write Eq. (4) using this information.

$$\frac{T_1}{T_0}=\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}} \qquad (5)$$

Combine Eqs. (3) and (5).

$$V_1=\sqrt{2c_P\left(T_0-T_1\right)}=\sqrt{2c_PT_0\left(1-\frac{T_1}{T_0}\right)}$$

$$\boxed{\therefore V_1=\sqrt{2c_PT_0\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}}\right]}} \qquad (6)$$

Using the given numerical data:

c_P = 1004.5 J/(kg·K) (for air)
γ = 1.4 (for air)
T_0 = 293 K
p_2/p_1 = 1.05
⇒ $\boxed{V_1 = 90\text{ m/s}}$

The mass flow rate can be found from the conditions at point 1:

$$\boxed{\dot{m} = \rho_1 V_1 A_1 = \left(\frac{p_1}{RT_1}\right) V_1 A_1} \quad (7)$$

where the ideal gas law has been used. The pressure at point 1 can be found from the blower pressure ratio and the fact that p_2 is equal to atmospheric pressure.

$$p_1 = \left(\frac{p_1}{p_2}\right) p_0 \quad (8)$$

The temperature at point 1 can be found using Eq. (5). Using the given numerical data:

p_0 = 100 kPa
p_2/p_1 = 1.05
p_1 = 95.2 kPa
T_0 = 293 K
T_1 = 289 K
R = 287 J/(kg·K) (for air)
V_1 = 90 m/s
A_1 = 0.01 m^2
⇒ $\boxed{\dot{m} = 1.03 \text{ kg/s}}$