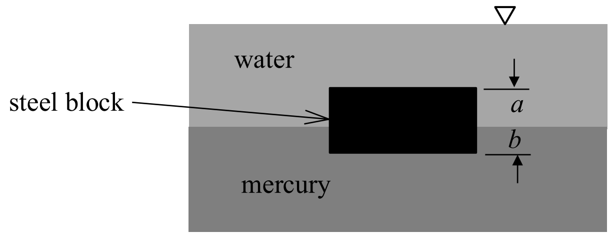
A uniform block of steel (with a specific gravity of 7.85) will “float” at a mercury-water interface as shown in the figure. What is the ratio of the distances *a* and *b*?



SOLUTION:

Balance forces in the vertical direction,

$$\sum F_V = 0 = -W_{\text{block}} + F_{B,Hg} + F_{B,H_2O} = -\rho_{\text{block}} V_{\text{block}} g + \rho_{Hg} V_{\substack{\text{block,}\\ \text{in } Hg}} g + \rho_{H_2O} V_{\substack{\text{block,}\\ \text{in } H_2O}} g\ , \tag{1}$$

where the buoyant forces are equal to the weights of the displaced fluids.

Re-writing in terms of the lengths a and b and the block's cross-sectional area A_{block},

$$-\rho_{\text{block}} A_{\text{block}} (a+b) + \rho_{Hg} A_{\text{block}} b + \rho_{H_2O} A_{\text{block}} a = 0\ , \tag{2}$$

$$-\rho_{\text{steel}} (a+b) + \rho_{Hg} b + \rho_{H_2O} a = 0\ , \tag{3}$$

$$-\rho_{H_2O} SG_{\text{steel}} b \left(\frac{a}{b} + 1 \right) + \rho_{H_2O} SG_{Hg} + \rho_{H_2O} b \frac{a}{b} = 0\ , \tag{4}$$

$$-SG_{\text{steel}} \left(\frac{a}{b} + 1 \right) + SG_{Hg} + \frac{a}{b} = 0\ , \tag{5}$$

$$\boxed{\frac{a}{b} = \frac{SG_{Hg} - SG_{\text{steel}}}{SG_{\text{steel}} - 1}}\ . \tag{6}$$

Using the given data,

SG_{Hg} = 13.6
SG_{steel} = 7.85
$\Rightarrow$ $\boxed{a/b = 0.83}$

Note that we could also solve this problem by balancing the block's weight with the pressure forces acting on the top and bottom block surfaces.

$$\sum F_V = 0 = -W_{\text{block}} + F_{p,H_2O} + F_{p,Hg} = -\rho_{\text{block}} A_{\text{block}} (a+b) g - \rho_{H_2O} g (H-a) A_{\text{block}} + \left(\rho_{H_2O} g H + \rho_{Hg} g b \right) A_{\text{block}}\ , \tag{7}$$

where H is the depth of the water-mercury interface. Simplifying this equation gives,

$$-\rho_{\text{block}} (a+b) - \rho_{H_2O} (H-a) + \rho_{H_2O} H + \rho_{Hg} b = 0\ , \tag{8}$$

$$-\rho_{\text{block}} (a+b) + \rho_{H_2O} a + \rho_{Hg} b = 0\ , \tag{9}$$

which is exactly the same as Eq. (3).