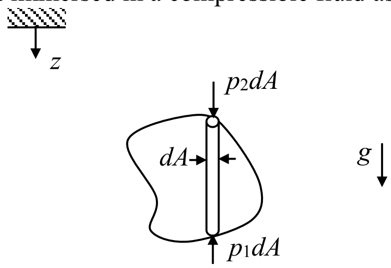
Archimedes principle states that the buoyant force acting on a submerged object is equal to the weight of the fluid displaced by that object. Is this true for *compressible* fluids?

SOLUTION:

Consider an arbitrary object immersed in a compressible fluid as shown in the figure below.




Determine the net pressure force acting on a parallelpiped of the material with a differential cross-sectional area,

$$dF_P = \left(p_1 - p_2\right) dA, \tag{1}$$

where,

$$p_1 = p_{z=0} + \int_{z=0}^{z=z_1} \rho g \, dz , \tag{2}$$

and,

$$p_2 = p_{z=0} + \int_{z=0}^{z=z_2} \rho g \, dz , \tag{3}$$

where $\rho$ is the density of the fluid (not the object).

Equation (1) becomes,

$$dF_P = \left( p_{z=0} + \int_{z=0}^{z=z_1} \rho g \, dz - p_{z=0} - \int_{z=0}^{z=z_2} \rho g \, dz \right) dA , \tag{4}$$

$$dF_P = dA \int_{z=z_2}^{z=z_1} \rho g dz . \tag{5}$$

The net pressure force acting over the entire object, i.e., the buoyant force, is,

$$F_P = \int_A dF_P = \int_A \int_{z=z_2}^{z=z_1} \rho g dz dA . \tag{6}$$

Assuming that the gravitational acceleration is constant (usually a good assumption),

$$F_P = g \int_A \int_{z=z_2}^{z=z_1} \rho \, dz dA , \tag{7}$$

Note that the integrals in the previous equation give the mass of the fluid displaced by the object, i.e.,

$$M_{\substack{\text{fluid displaced}\\ \text{by object}}} = \int_A \int_{z=z_2}^{z=z_1} \rho \, dz dA . \tag{8}$$

Thus, just as with the incompressible case, <u>the buoyant force in a compressible fluid is equal to the weight of the fluid displaced by the object</u>,

$$F_p = M_{\substack{\text{fluid displaced}\\ \text{by object}}} g . \tag{9}$$