A Venturi pump is used in the design of a carburetor, a device used to create a fuel-air mixture to be fed into the cylinder of an internal combustion engine. Simplified schematics of a carburetor are shown in the following figures.

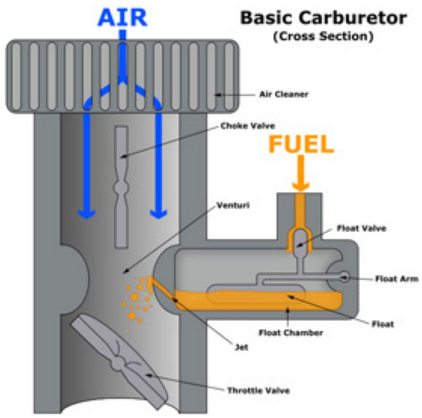
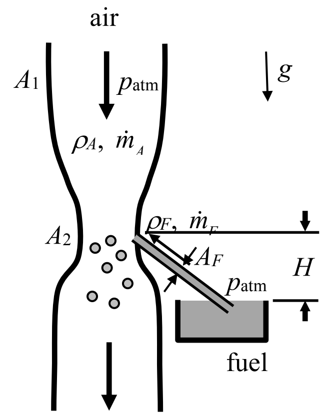


Image from: http://hdabob.com/wp-content/uploads/2009/10/carburetor.jpg

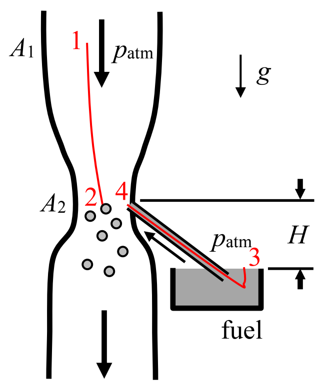


The air, which may reasonably be assumed to be incompressible, has a density ρ_A and the liquid fuel has density ρ_F. The fuel reservoir is located a distance H below the inlet port into the Venturi. The inlet air is at atmospheric pressure as is the free surface of the fuel reservoir. The air inlet cross-sectional area is A_1 and the Venturi throat area is A_2. The fuel line cross-sectional area is A_F.

If the desired air-to-fuel mass flow rate ratio at the outlet of the carburetor is R (= $\dot{m}_A / \dot{m}_F$), determine the required ratio A_1/A_2 in terms of (a subset of) the air-to-fuel ratio R, air density ρ_A, the fuel density ρ_F, the inlet air mass flow rate $\dot{m}_A$, the acceleration due to gravity g, the height from the fuel reservoir to the Venturi throat H, the fuel pipe area A_F, and the air inlet area A_1.

SOLUTION:

Apply Bernoulli's equation from 1 to 2.



$$\left(\frac{p}{\rho_A g}+\frac{V^2}{2g}+z\right)_2=\left(\frac{p}{\rho_A g}+\frac{V^2}{2g}+z\right)_1 \tag{1}$$

where

$$p_1 = p_{atm} \text{ and } p_2 = ? \tag{2}$$

$$V_1=\frac{\dot{m}_A}{\rho_A A_1} \text{ and } V_2=\frac{\dot{m}_A}{\rho_A A_2} \tag{3}$$

Δz is negligible compared to the other terms in B.E. since the fluid is a gas (4)

Substitute and simplify.

$$p_2-p_{atm}=\frac{1}{2}\frac{\dot{m}_A^2}{\rho_A}\left(\frac{1}{A_1^2}-\frac{1}{A_2^2}\right) \tag{5}$$

Apply Bernoulli's equation from 3 to 4.

$$\left(\frac{p}{\rho_F g}+\frac{V^2}{2g}+z\right)_4=\left(\frac{p}{\rho_F g}+\frac{V^2}{2g}+z\right)_3 \tag{6}$$

where

$$p_3 = p_{atm} \text{ and } p_4 = p_2 \tag{7}$$

$$V_3 \approx 0 \text{ and } V_4=\frac{\dot{m}_F}{\rho_F A_F} \tag{8}$$

$$\Delta z = z_4 - z_3 = H \tag{9}$$

Substitute and simplify.

$$\underbrace{p_4}_{=p_2}-p_{atm}=-\rho_F gH-\frac{1}{2}\frac{\dot{m}_F^2}{\rho_F A_F^2} \tag{10}$$

Combine Eqs. (5) and (10) and solve for A_1/A_2.

$$\frac{1}{2}\frac{\dot{m}_A^2}{\rho_A}\left(\frac{1}{A_1^2}-\frac{1}{A_2^2}\right)=-\rho_F gH-\frac{1}{2}\frac{\dot{m}_F^2}{\rho_F A_F^2} \tag{11}$$

$$\left(\frac{1}{A_1^2}-\frac{1}{A_2^2}\right)=-2\rho_A\rho_F\frac{gH}{\dot{m}_A^2}-\frac{1}{A_F^2}\frac{\rho_A}{\rho_F}\frac{\dot{m}_F^2}{\dot{m}_A^2} \tag{12}$$

$$\left(1-\frac{A_1^2}{A_2^2}\right)=-2\rho_A\rho_F\frac{gHA_1^2}{\dot{m}_A^2}-\frac{A_1^2}{A_F^2}\frac{\rho_A}{\rho_F}\frac{\dot{m}_F^2}{\dot{m}_A^2} \tag{13}$$

$$\left(\frac{A_1}{A_2}\right)^2=1+2\rho_A\rho_F\frac{gHA_1^2}{\dot{m}_A^2}+\left(\frac{A_1}{A_F}\right)^2\left(\frac{\rho_A}{\rho_F}\right)\left(\frac{\dot{m}_F}{\dot{m}_A}\right)^2 \tag{14}$$

$$\boxed{\frac{A_1}{A_2}=\sqrt{1+2\rho_A\rho_F\frac{gHA_1^2}{\dot{m}_A^2}+\left(\frac{A_1}{A_F}\right)^2\left(\frac{\rho_A}{\rho_F}\right)\left(\frac{\dot{m}_F}{\dot{m}_A}\right)^2}} \text{ or } \boxed{\frac{A_1}{A_2}=\sqrt{1+2\rho_A\rho_F\frac{gHA_1^2}{\dot{m}_A^2}+\left(\frac{A_1}{A_F}\right)^2\left(\frac{\rho_A}{\rho_F}\right)\frac{1}{R^2}}} \tag{15}$$

For a typical carburetor,

ρ_F = 770 kg/m^3 (gasoline)
ρ_A = 1.23 kg/m^3 (air)
A_1 = 1.34*10^{-3} m^2 (D_1 = 4.13cm = 1 5/8 in.)
A_F = 1.70*10^{-6} m^2 (D_F = 1.47 mm = 0.058 in.)
R = 14.7 (ideal fuel to air ratio for gasoline)
g = 9.81 m/s^2
H = 2.00*10^{-2} m (= 2 cm)
$\dot{m}_A$ = 0.290 kg/s (500 cfm @ 1.23 kg/m^3)
$\Rightarrow$ A_1/A_2 = 2.36 $\Rightarrow$ D_2 = 2.69 cm