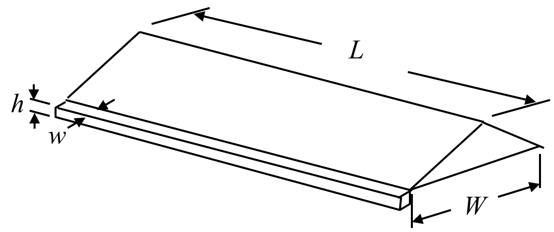
Consider a simple roof with the geometry shown below.



Assume that on each side of the house there is a gutter running the length of the house. The gutter has a width of w. During a rainstorm in which rain falls straight down at a rate of r (in depth per hour, *e.g.* 1 in. per hour), estimate the rate at which the depth in the gutter increases if the gutter exit is clogged. Assume that no water accumulates on the roof, *i.e.* the water landing on the roof immediately drains into the gutter.

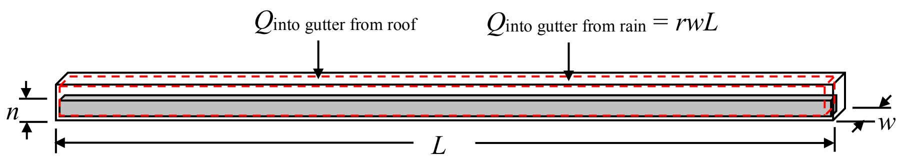
If $L = 100$ ft, $W = 40$ ft, $w = 3.5$ in., $h = 3.5$ in., and $r = 1$ in./hr estimate how long it will take before the water in the gutter spills out.

SOLUTION:

Since no water accumulates on the roof, all of the rain that lands on the roof flows directly into the gutter. Hence, the flow rate into the gutter due to the water from the roof is:

$$Q_{\substack{\text{into gutter}\\\text{from roof}}} = r\tfrac{1}{2}WL \qquad (1)$$

Apply conservation of mass to a control volume located just within the gutter as shown in the following figure.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = 0 \qquad (2)$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{d}{dt}(\rho wLn) = \rho wL\frac{dn}{dt} \qquad (3)$$

$$\int_{CS}\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = -\rho Q_{\substack{\text{into gutter}\\\text{from roof}}} - \rho Q_{\substack{\text{into gutter}\\\text{from rain}}} = -\rho r\tfrac{1}{2}WL - \rho rwL \qquad (4)$$

Substitute and solve for dn/dt.

$$\rho wL\frac{dn}{dt} - \rho r\tfrac{1}{2}WL - \rho rwL = 0 \qquad (5)$$

$$\boxed{\therefore \frac{dn}{dt} = r\left(\frac{W}{2w}+1\right)} \Rightarrow \boxed{n = r\left(\frac{W}{2w}+1\right)t} \text{ (since } n(t=0)=0\text{)} \qquad (6)$$

Using the given parameters:

r = 1 in./hr = 1.67×10^{-2} in./min
W = 40 ft = 480 in.
w = 3.5 in.
h = 3.5 in.
$\boxed{t_{\text{fill}} = 3.0 \text{ min.}}$