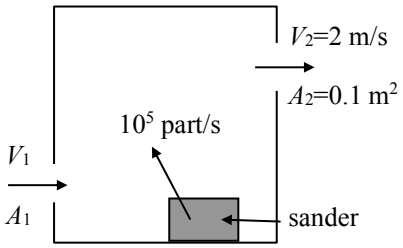
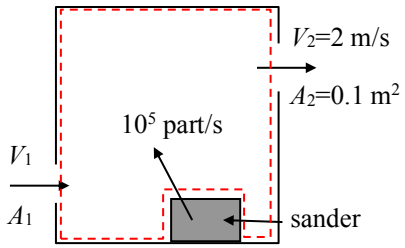
A sanding operation injects 10^5 particles/s into the air in a room as shown in the figure. The amount of dust in the room is maintained at a constant level by a ventilating fan that draws clean air into the room at section 1 and expels dusty air at section 2. Determine the concentration of particles (particles/m^3) in the exhaust air for steady state conditions.



SOLUTION:

Apply conservation of mass to the control volume shown below.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = \mu C_2 V_2 A_2 - \mu C_1 V_1 A_1 - \mu N \tag{3}$$

where μ is the mass of a dust particle (with dimensions of mass per particle), C is the concentration of dust particles (particles per unit volume), and N is the number flux of particles generated by the sander (particles per unit time).

The incoming air has no dust particles so $C_1 = 0$. The outgoing air will have the same concentration of dust particles as the air within the room. Simplify Eqn. (1) using Eqns. (2) and (3) gives:

$$\mu C_2 V_2 A_2 - \mu C_1 V_1 A_1 - \mu N = 0 \tag{4}$$

$$C_2 = \frac{N}{V_2 A_2} \tag{5}$$

Using the given numerical data:

V_2 = 2 m/s

A_2 = 0.1 m^2

N = 10^5 part/s

$\Rightarrow$ $\boxed{C_2 = 5*10^5 \text{ part/m}^3}$