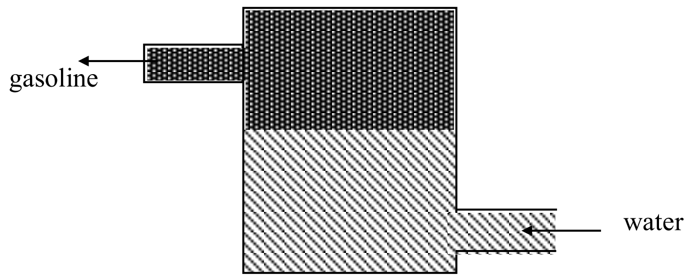
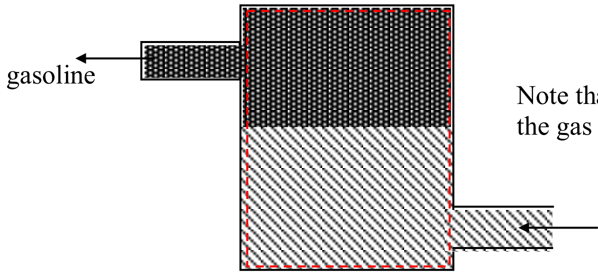
Water enters a rigid, sealed, cylindrical tank at a steady rate of 100 L/hr and forces gasoline (with a specific gravity of 0.68) out as is indicated in the drawing. The tank has a total volume of 1000 L. What is the time rate of change of the mass of gasoline contained in the tank?



SOLUTION:

Apply conservation of mass to the control volume shown below.



Note that the interface between the gas and water is moving.

$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{d}{dt}\left(M_{gas} + \underbrace{M_{H20}}_{=\rho_{H20}V_{H20}}\right) = \frac{dM_{gas}}{dt} + \rho_{H20}\frac{dV_{H20}}{dt}$$ (Gas and water are incompressible.)

$$\int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = \rho_{gas}Q_{gas} - \rho_{H20}Q_{H20}$$

Substitute and simplify.

$$\frac{dM_{gas}}{dt} + \rho_{H20}\frac{dV_{H20}}{dt} + \rho_{gas}Q_{gas} - \rho_{H20}Q_{H20} = 0$$

$$\frac{dM_{gas}}{dt} = \rho_{H20}\left(Q_{H20} - \frac{dV_{H20}}{dt}\right) - \rho_{gas}Q_{gas} \tag{2}$$

Note that the time rate of change of the water volume, dV_{H20}/dt, is equal to the water's volumetric flow rate, Q_{H20}. Furthermore, since both liquids are incompressible and the total tank volume remains constant, $Q_{gas} = Q_{H20}$. Utilizing these facts to simply Eqn. (2) gives:

$$\boxed{\frac{dM_{gas}}{dt} = -\rho_{gas}Q_{H20} = -SG_{gas}\rho_{H20}Q_{H20}} \tag{3}$$

Using the given parameters:

SG_{gas} = 0.68

ρ_{H20} = 1000 kg/m^3

Q_{H20} = 100 L/hr = 0.1 m^3/hr

$\Rightarrow$ $\boxed{dM_{gas}/dt = \text{-68 kg/hr} = 0.019\text{ kg/s}}$