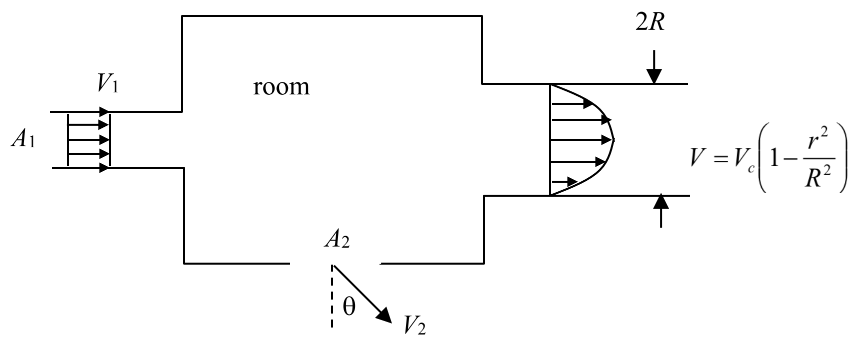
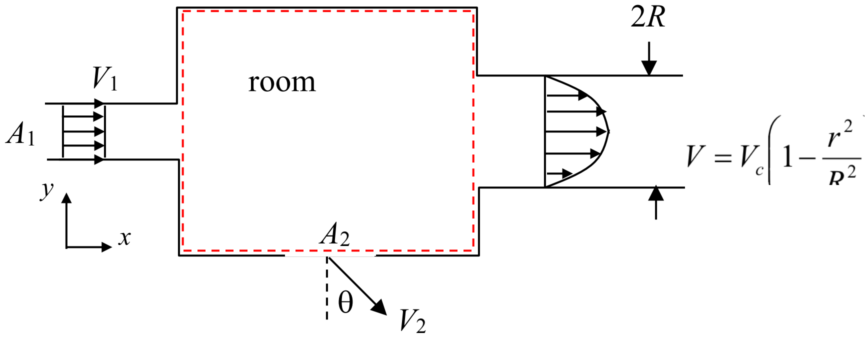
Determine the rate at which fluid mass collects inside the room shown below in terms of ρ, V_1, A_1, V_2, A_2, V_c, R, and θ. Assume the fluid moving through the system is incompressible.



SOLUTION:

Apply conservation of mass to a control volume fixed to the interior of the room.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = 0 \qquad (1)$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{dM_{CV}}{dt}$$

$$\int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = \rho\left(V_1\hat{\mathbf{i}}\cdot -A_1\hat{\mathbf{i}}\right)+\rho\left[\left(V_2\sin\theta\hat{\mathbf{i}}-\cos\theta\hat{\mathbf{j}}\right)\cdot -A_2\hat{\mathbf{j}}\right]+\rho\int_{r=0}^{r=R}V\hat{\mathbf{i}}\cdot dA\hat{\mathbf{i}}$$

$$= -\rho V_1 A_1 + \rho V_2 A_2\cos\theta + \rho\int_{r=0}^{r=R}V_c\left(1-r^2/R^2\right)2\pi r dr$$

$$= -\rho V_1 A_1 + \rho V_2 A_2\cos\theta + \tfrac{\pi}{2}\rho V_c R^2$$

Substitute and simplify.

$$\frac{dM_{CV}}{dt} - \rho V_1 A_1 + \rho V_2 A_2\cos\theta + \tfrac{\pi}{2}\rho V_c R^2 = 0$$

$$\boxed{\therefore \frac{dM_{CV}}{dt} = \rho V_1 A_1 - \rho V_2 A_2\cos\theta - \tfrac{\pi}{2}\rho V_c R^2} \qquad (2)$$