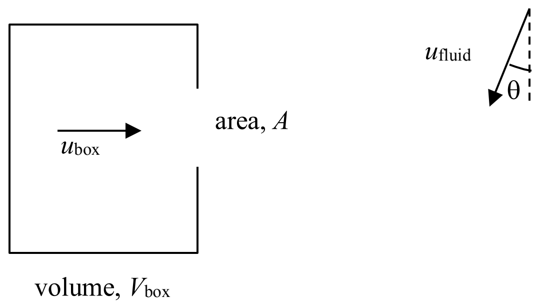
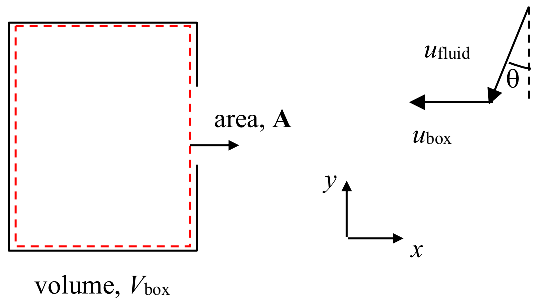
A box with a hole of area, A, moves to the right with velocity, u_{box}, through an incompressible fluid as shown in the figure. If the fluid has a velocity of u_{fluid} which is at an angle, θ, to the vertical, determine how long it will take to fill the box with fluid. Assume the box volume is V_{box} and that it is initially empty.



SOLUTION:

Apply conservation of mass to a control volume fixed to the interior of the box. Change our frame of reference so the box appears stationary.



$$\frac{d}{dt}\int_{\text{CV}}\rho dV + \int_{\text{CS}}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = 0 \quad (1)$$

where

$$\frac{d}{dt}\int_{\text{CV}}\rho dV = \frac{d\left(\rho V_{\text{CV}}\right)}{dt} = \rho\frac{dV_{\text{CV}}}{dt}$$

$$\int_{\text{CS}}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = \rho\left[\underbrace{\left(-u_{\text{box}}\hat{\mathbf{i}} - u_{\text{fluid}}\sin\theta\hat{\mathbf{i}} - u_{\text{fluid}}\cos\theta\hat{\mathbf{j}}\right)}_{=\mathbf{u}_{\text{rel}}}\cdot\underbrace{A\hat{\mathbf{i}}}_{=d\mathbf{A}}\right] = -\rho\left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)A$$

Substitute and simplify.

$$\rho\frac{dV_{\text{CV}}}{dt} - \rho\left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)A = 0$$

$$\frac{dV_{\text{CV}}}{dt} = \left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)A$$

$$\int_{V_{\text{CV}}=0}^{V_{\text{CV}}=V_{\text{CV}}} dV_{\text{CV}} = \left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)A\int_{t=0}^{t=T} dt$$ (Note that u_{box}, u_{fluid}, θ, and A don't change with time.)

$$V_{\text{CV}} = \left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)AT$$

$$\boxed{\therefore T = \frac{V_{\text{CV}}}{\left(u_{\text{box}} + u_{\text{fluid}}\sin\theta\right)A}} \quad (2)$$