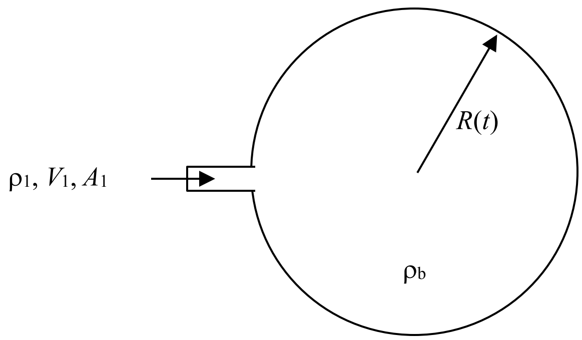
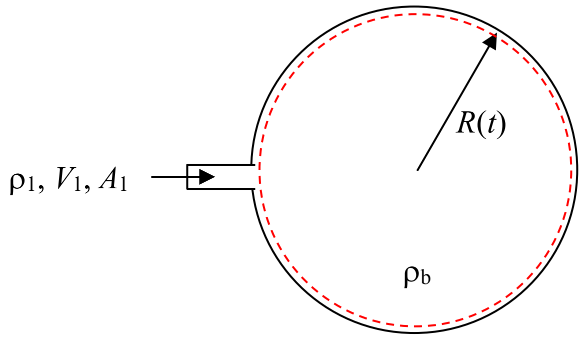
A spherical balloon is filled through an area, A_1, with air flowing at velocity, V_1, and constant density, ρ_1. The radius of the balloon, $R(t)$, can change with time, t. The average density within the balloon at any given time is $\rho_b(t)$. Determine the relationship between the rate of change of the density within the balloon and the rest of the variables.



SOLUTION:

Apply conservation of mass to a control volume that deforms to follow the interior surface of the balloon.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{d}{dt}\Big(\underbrace{\rho_b \frac{4}{3}\pi R^3}_{=M_{CV}}\Big) = \frac{4}{3}\pi R^3 \frac{d\rho_b}{dt} + 4\pi\rho_b R^2 \frac{dR}{dt}$$

$$\int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = -\rho_1 V_1 A_1$$

Substitute and simplify.

$$\frac{4}{3}\pi R^3 \frac{d\rho_b}{dt} + 4\pi\rho_b R^2 \frac{dR}{dt} - \rho_1 V_1 A_1 = 0$$

$$\boxed{\frac{d\rho_b}{dt} = \frac{\rho_1 V_1 A_1 - 4\pi\rho_b R^2 \frac{dR}{dt}}{\frac{4}{3}\pi R^3}} \tag{2}$$