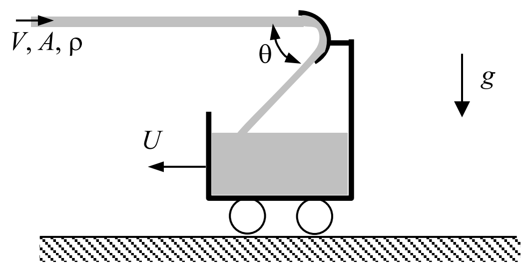
A cart travels at velocity, $U$, toward a liquid jet that has a velocity, $V$, relative to the ground, a density, ρ, and a constant area, $A$. The mass of the cart and its contents at time $t = 0$ is $M_0$ and the cart's initial velocity is $U_0$ toward the jet. The resistance between the cart's wheels and the surface is negligible.

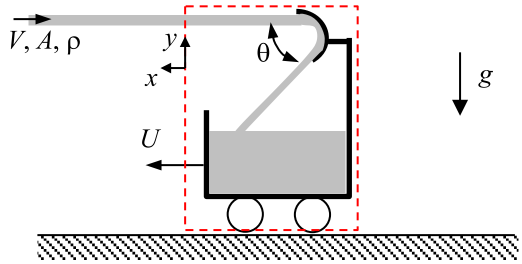



a. Determine the mass flow rate into the cart in terms of (a subset of) $\rho$, $A$, $V$, $U$, $g$, and $\theta$.

b. Determine the acceleration of the cart, $dU/dt$, in terms of (a subset of) $\rho$, $A$, $V$, $U$, $g$, $\theta$, and $M(t)$ where $M(t)$ is the mass of the cart and water at time $t$. You needn't solve any integrals or differential equations that appear in your answer.

SOLUTION:

Apply conservation of mass to a control volume surrounding the cart.




$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{dM}{dt} \tag{2}$$

$$\int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = -\rho\left(U+V\right)A \tag{3}$$

Note that the rate at which liquid mass *enters* the CV is $\boxed{\dot{m}_{\substack{\text{into}\\\text{cart}}} = -\int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = \rho\left(U+V\right)A}$ (4)

Substitute and simplify.

$$\frac{dM}{dt} - \rho\left(U+V\right)A = 0 \tag{5}$$

$$\frac{dM}{dt} = \rho\left(U+V\right)A \tag{6}$$

Note that $U = U(t)$.

Now apply the linear momentum equation in the $x$-direction to the same control volume. Use a frame of reference fixed to the cart (non-inertial).

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV + \int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} - \int_{\text{CV}} a_{x/X} \rho dV \qquad (7)$$

where

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV \approx 0 \quad \text{(Most of the material in the CV has zero horz. velocity in this FOR.)} \qquad (8)$$

$$\int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \left[-\left(U+V\right)\right]\left[-\rho\left(U+V\right)A\right] = \rho\left(U+V\right)^2 A \qquad (9)$$

$$F_{B,x} = 0 \qquad (10)$$

$$F_{S,x} = 0 \qquad (11)$$

$$\int_{\text{CV}} a_{x/X} \rho dV = \frac{dU}{dt} M \qquad (12)$$

Substitute and simplify.

$$\rho\left(U+V\right)^2 A = -\frac{dU}{dt} M \qquad (13)$$

$$\boxed{\frac{dU}{dt} = -\frac{\rho\left(U+V\right)^2 A}{M}} \qquad (14)$$

Note that $M = M(t)$ and $U = U(t)$. To solve for the motion of the cart, one would need to solve Eqns. (6) and (14) simultaneously subject to the initial conditions $M(t = 0) = M_0$ and $U(t = 0) = U_0$.