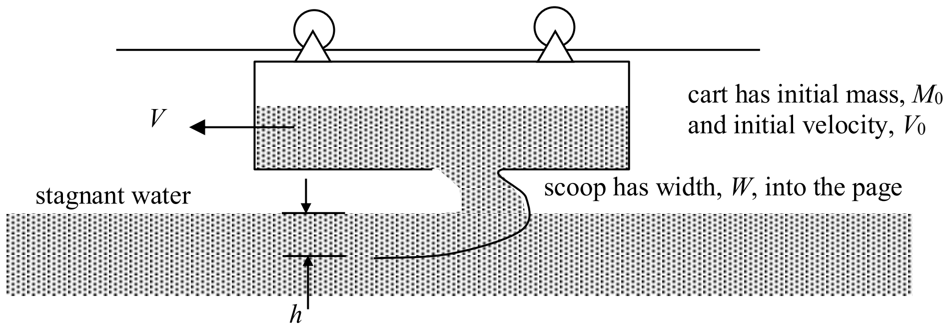
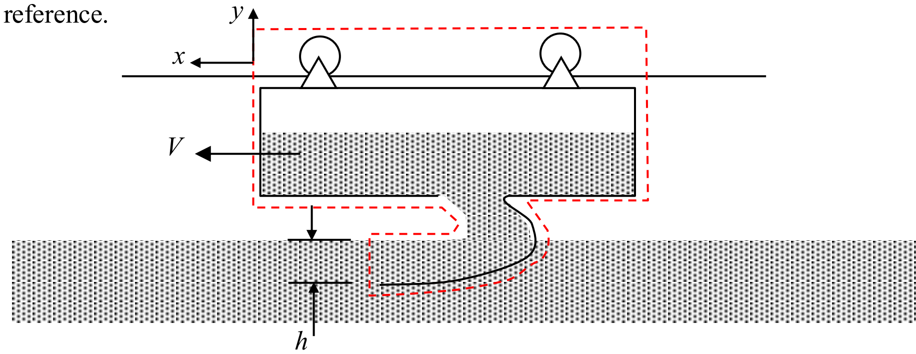
A cart hangs from a wire as shown in the figure below. Attached to the cart is a scoop of width *W* (into the page) which is submerged into the water a depth, *h*, from the free surface. The scoop is used to fill the cart tank with water of density, ρ.



a. Show that at any instant $V = V_0 M_0 / M$ where *M* is the mass of the cart and the fluid within the cart.
b. Determine the velocity, *V*, as a function of time.

SOLUTION:

Apply the linear momentum equation in the *x*-direction to the control volume shown using the indicated frame of reference.



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x\left(\rho \mathbf{u}_{rel}\cdot d\mathbf{A}\right) = F_{S,x} + F_{B,x} - \int_{CV} a_{x/X}\rho dV$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV \approx 0$$

(The *x*-linear momentum within the CV is approximately zero in the given frame of reference.)

$$\int_{CS} u_x\left(\rho \mathbf{u}_{rel}\cdot d\mathbf{A}\right) = -V\rho\left(-V\right)hW = \rho V^2 hW$$

$F_{S,x} = 0$ (The pressure forces on the front and rear portions of the scoop cancel each other out.)

$F_{B,x} = 0$

$$\int_{CV} a_{x/X}\rho dV = \frac{dV}{dt}M$$

Substitute and simplify:

$$\rho V^2 hW = -\frac{dV}{dt}M \tag{1}$$

Apply conservation of mass to the same control volume in order to determine the mass as a function of time.

$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = 0$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{dM}{dt}$$

$$\int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = -\rho VhW$$

Substitute and simplify:

$$\frac{dM}{dt} - \rho VhW = 0$$

$$\frac{dM}{dt} = \rho VhW \tag{2}$$

Substitute Eqn. (2) into Eqn. (1):

$$\frac{dM}{dt}V = -\frac{dV}{dt}M$$

$$\int_{M_0}^{M}\frac{dM}{M} = -\int_{V_0}^{V}\frac{dV}{V}$$

$$\ln\frac{M}{M_0} = -\ln\frac{V}{V_0} \Rightarrow \frac{M}{M_0} = \frac{V_0}{V}$$

$$\ln\frac{M}{M_0} = -\ln\frac{V}{V_0} \Rightarrow \frac{M}{M_0} = \frac{V_0}{V}$$

$$\boxed{\therefore V = V_0\frac{M_0}{M}} \tag{3}$$

To determine the cart velocity as a function of time, combine Eqns. (1) and (3):

$$\rho V^2 hW = -\frac{dV}{dt}\frac{V_0}{V}M_0$$

$$\int_0^t dt = -\frac{M_0 V_0}{\rho hW}\int_{V_0}^{V}\frac{dV}{V^3}$$

$$t = \frac{M_0 V_0}{2\rho hW}\left(\frac{1}{V^2} - \frac{1}{V_0^2}\right) \Rightarrow V = \left(\frac{2\rho hWt}{M_0 V_0} + \frac{1}{V_0^2}\right)^{-1/2}$$

$$\boxed{\frac{V}{V_0} = \frac{1}{\sqrt{\frac{2\rho hWV_0 t}{M_0} + 1}}} \tag{4}$$