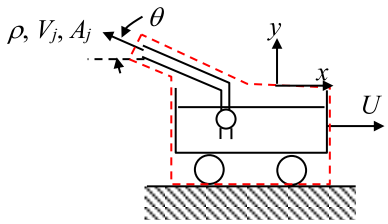
A cart with frictionless wheels holds a water tank, motor, pump, and nozzle. The cart is on horizontal ground and initially still. At time zero the cart has a mass M_0 and the pump is started to produce a jet of water with constant area A_j, velocity V_j at an angle θ with respect to the horizontal. Find and solve the equations governing the mass and velocity of the cart as a function of time.

SOLUTION:

Apply the linear momentum equation in the x-direction to a control volume surrounding the cart. Use a frame of reference fixed to the control volume (non-inertial).



The frame of reference is fixed to the (accelerating) control volume and, hence, is non-inertial.

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV + \int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} - \int_{\text{CV}} a_{x/X} \rho dV$$

where

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV \approx 0$$

(Using the given FOR, the rate of change of the CV linear momentum is nearly zero since most of the mass in the CV has a constant (=0) horizontal velocity.)

$$\int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \left(-V_j \cos\theta\right)\left(\rho V_j A_j\right) = -\rho V_j^2 A_j \cos\theta$$

$$F_{B,x} = 0$$

$$F_{S,x} = 0$$

$$\int_{\text{CV}} a_{x/X} \rho dV = M_{\text{CV}} \frac{dU}{dt}$$ (Note that the CV mass changes with time.)

Substitute and solve for the cart acceleration.

$$\frac{dU}{dt} = \frac{\rho V_j^2 A_j \cos\theta}{M_{\text{CV}}} \tag{1}$$

Determine the mass inside the control volume using conservation of mass applied to the same control volume.

$$\frac{d}{dt}\int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0$$

where

$$\frac{d}{dt}\int_{\text{CV}} \rho dV = \frac{dM_{\text{CV}}}{dt}$$

$$\int_{\text{CS}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = \rho V_j A_j$$

Substitute and solve for M_{CV}.

$$\frac{dM_{CV}}{dt} = -\rho V_j A_j$$

$$\int_{M_0}^{M_{CV}} dM_{CV} = -\rho V_j A_j \int_0^t dt$$ (Note that $\rho V_j A_j$ is constant with respect to time.)

$$\boxed{M_{CV} = M_0 - \rho V_j A_j t} \tag{2}$$

Substitute Eqn. (2) into Eqn. (1) and solve for U.

$$\frac{dU}{dt} = \frac{\rho V_j^2 A_j \cos\theta}{M_0 - \rho V_j A_j t}$$

$$\int_0^U dU = \int_0^t \frac{\rho V_j^2 A_j \cos\theta dt}{M_0 - \rho V_j A_j t}$$

$$U = \frac{\rho V_j^2 A_j \cos\theta}{-\rho V_j A_j} \ln\left(\frac{M_0 - \rho V_j A_j t}{M_0}\right)$$

$$\boxed{\therefore U = -V_j \cos\theta \ln\left(1 - \frac{\rho V_j A_j t}{M_0}\right)} \tag{3}$$