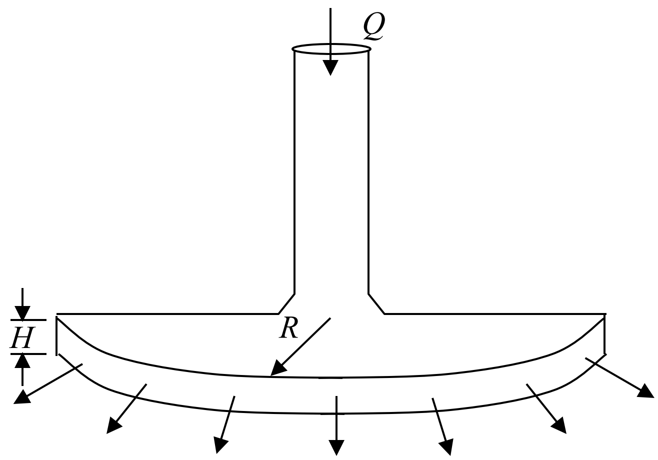
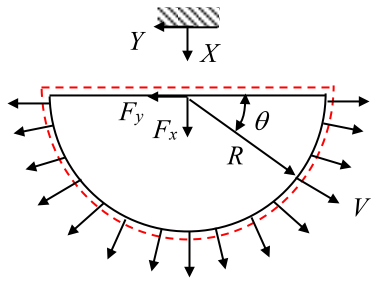
Water is sprayed radially outward through 180° as shown in the figure. The jet sheet is in the horizontal plane and has thickness, H. If the jet volumetric flow rate is Q, determine the resultant horizontal anchoring force required to hold the nozzle stationary.

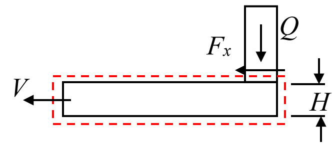


SOLUTION:

Apply the linear momentum equation in the X direction to the fixed control volume shown below.



top view



side view

$$\frac{d}{dt}\int_{CV} u_X \rho dV + \int_{CS} u_X\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,X} + F_{S,X} \tag{1}$$

where

$$\frac{d}{dt}\int_{CV} u_X \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{CS} u_X\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \int_{\theta=0}^{\theta=\pi} \overbrace{\left(V\sin\theta\right)}^{=u_X}\left(\overbrace{\rho V\, R d\theta H}^{=dA}\right) = \rho V^2 RH \int_{\theta=0}^{\theta=\pi} \sin\theta d\theta = -\rho V^2 RH\cos\theta\Big|_0^{\pi}$$

$$= -\rho V^2 RH\left(-1-1\right)$$

$$= 2\rho V^2 RH$$

(Note that there is no X-momentum at the control volume inlet. Also, V is an unknown quantity at the moment.)

$F_{B,X} = 0$

$F_{S,X} = F_x$ (All of the pressure forces cancel and only the anchoring force remains.)

Substitute.

$$F_x = 2\rho V^2 RH \tag{2}$$

To determine V, apply conservation of mass to the same control volume.

$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0 \tag{3}$$

where

$$\frac{d}{dt}\int_{CV} \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{CS} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = \underbrace{-\rho Q}_{\text{inlet}} + \underbrace{\int_{\theta=0}^{\theta=\pi} \rho V\, \overbrace{R d\theta H}^{=dA}}_{\text{outlet}} = -\rho Q + \rho V \pi RH$$

Substitute and simplify.

$$-\rho Q + \rho V \pi RH = 0$$

$$V = \frac{Q}{\pi RH} \tag{4}$$

Substitute Eqn. (4) into Eqn. (2).

$$\boxed{F_x = 2\rho\left(\frac{Q}{\pi RH}\right)^2 RH} \tag{5}$$

Note that $F_y = 0$ due to symmetry.