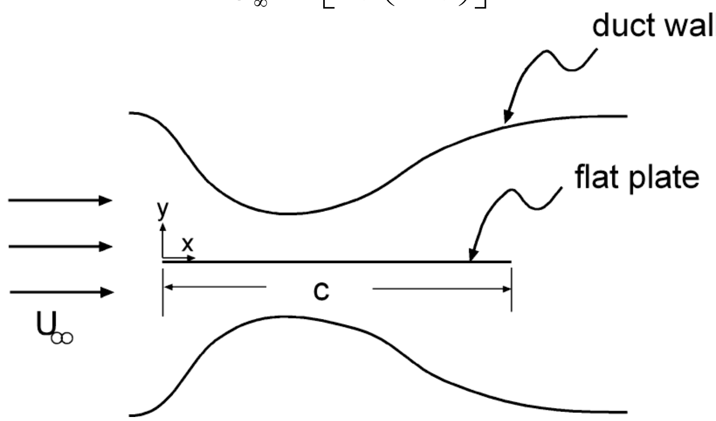
A flat plate of length c is placed inside a duct. By curving the walls of the duct, the pressure distribution on the flat plate can be set. Assume the walls of the duct are contoured in such a way that the outer flow over the plate gives the following velocity on the surface of the flat plate:

$$\frac{u_e(x)}{U_\infty} = \left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5}$$



1. Write an expression for the streamwise pressure gradient as a function of x/c.
2. Determine which portions of the plate have a favorable pressure gradient and which portions have an adverse pressure gradient.

SOLUTION:

In the outer flow region (the inviscid core), we can use Bernoulli's equation,

$$p+\tfrac{1}{2}\rho U^2=\text{constant} \;\Rightarrow\; \frac{dp}{dx}+\rho U\frac{dU}{dx}=0 \;\Rightarrow\; \frac{dp}{dx}=-\rho U\frac{dU}{dx} \tag{1}$$

Here,

$$U=u_e\left(x\right)=U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5} \;\Rightarrow\; \frac{dU}{dx}=\frac{8}{5}U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(\frac{1}{c}-\frac{2x}{c^2}\right) \;\Rightarrow$$

$$\frac{dU}{dx}=\frac{8}{5}\frac{U_\infty}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(1-2\frac{x}{c}\right) \tag{2}$$

Thus,

$$\frac{dp}{dx}=-\rho\left\{U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5}\right\}\left\{\frac{8}{5}\frac{U_\infty}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(1-2\frac{x}{c}\right)\right\} \tag{3}$$

$$\boxed{\frac{dp}{dx}=-\frac{8}{5}\frac{\rho U_\infty^2}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{3}{5}}\left(1-2\frac{x}{c}\right)} \tag{4}$$

An adverse pressure gradient is one in which $dp/dx > 0$. A favorable pressure gradient is one in which $dp/dx < 0$. Note also that $0 \le x/c \le 1$.

$$-\frac{8}{5}\frac{\rho U_\infty^2}{c}\underbrace{\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{3}{5}}}_{>0\text{ for }0<x/c<1}\underbrace{\left(1-2\frac{x}{c}\right)}_{>0\text{ for }x/c<\frac{1}{2}}\begin{cases}<0 & x/c<\frac{1}{2}\\ >0 & x/c>\frac{1}{2}\end{cases} \tag{5}$$

Thus, there is a favorable pressure gradient for $x/c < ½$ and adverse pressure gradient for $x/c > ½$.