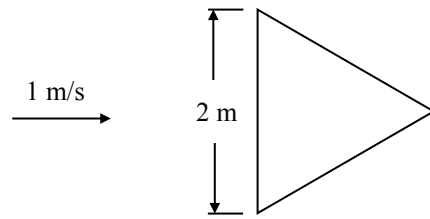
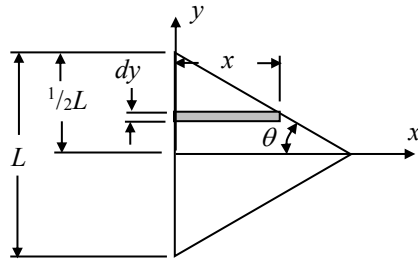


A thin equilateral triangle plate is immersed parallel to a 1 m/s stream of air at standard conditions. Estimate the skin friction drag on this plate.



SOLUTION:

Determine the drag acting on thin strips as shown in the figure below. The total drag will be the sum of each of these individual drag forces.



Since the plate is symmetric about the x -axis, consider only the $y > 0$ portion of the plate. The equation of the line defining the downstream edge of the plate is:

$$y = (-\tan \theta)x + \frac{1}{2}L \quad \text{where } 0 \leq x \leq L \cos \theta \quad (L = 2 \text{ m}, \theta = 30^\circ) \quad (1)$$

Re-arrange to solve for x in terms of y :

$$x = \frac{\frac{1}{2}L - y}{\tan \theta} \quad (2)$$

Determine if the flow transitions to turbulence at any point on the plate.

$$\text{Re}_{\text{crit}} = \frac{Ux_{\text{crit}}}{\nu} = 500,000 \Rightarrow x_{\text{crit}} = 500,000 \frac{\nu}{U} \quad (3)$$

Using the given data:

$$\nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$$

$$U = 1 \text{ m/s}$$

$$\Rightarrow x_{\text{crit}} = 7.5 \text{ m}$$

Since the longest strip length is only $L \cos \theta = (2 \text{ m}) \cos(30^\circ) = 1.73 \text{ m}$, the flow over the entire plate will be laminar. Hence, we can use the Blasius solution relations to model the boundary layer characteristics. The drag acting on a single strip of length x and width dy is:

$$\begin{aligned} dD &= c_D \frac{1}{2} \rho U^2 x dy \\ &= \frac{1.328}{\text{Re}_x^{1/2}} \frac{1}{2} \rho U^2 x dy \end{aligned} \quad (4)$$

$$\therefore dD = 0.664 \sqrt{\nu} \rho U^{3/2} x^{1/2} dy \quad (5)$$

The total drag acting on the plate (including the top half, bottom half, and front and back surfaces) is:

$$\begin{aligned} D &= 4 \int_{y=0}^{y=1/2 L} dD = 4 \int_{y=0}^{y=1/2 L} 0.664 \sqrt{\nu} \rho U^{3/2} x^{1/2} dy \\ &= 4 \int_{y=0}^{y=1/2 L} 0.664 \sqrt{\nu} \rho U^{3/2} \left(\frac{\frac{1}{2}L - y}{\tan \theta} \right)^{1/2} dy \end{aligned} \quad (\text{making use of Eqn. (2)}) \quad (6)$$

$$\boxed{\therefore D = 0.626 \rho (UL)^{3/2} \sqrt{\frac{\nu}{\tan \theta}}} \quad (7)$$

Using the given data:

$$\rho = 1.23 \text{ kg/m}^3$$

$$U = 1 \text{ m/s}$$

$$L = 2 \text{ m}$$

$$\nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\theta = 30^\circ$$

$$\Rightarrow \boxed{D = 1.1 \times 10^{-2} \text{ N}}$$