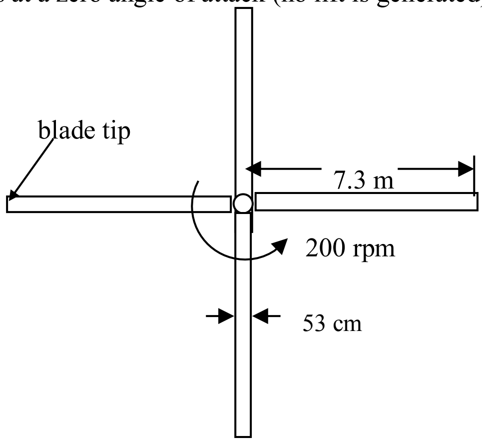
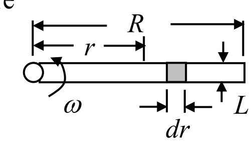
A four-bladed Apache helicopter rotor rotates at 200 rpm in air (with a density of 1.2 kg/m^3 and kinematic viscosity $1.5*10^{-5}$ m^2/s). Each blade has a chord length of 53 cm and extends a distance of 7.3 m from the center of the rotor hub. To greatly simplify the problem, assume that the blades can be modeled as very thin flat plates at a zero angle of attack (no lift is generated).



a. At what radial distance from the hub center is the flow at the blade trailing edge turbulent?
b. What is the (99%) boundary layer thickness at the blade tip trailing edge?
c. Assuming that the flow over the entire length of the four blades is turbulent, estimate the power required to drive the helicopter rotor (neglecting all other effects besides aerodynamic drag).

SOLUTION:

The transition to turbulence occurs when $\mathrm{Re}_{\mathrm{crit}}$ = 500,000 where



$$\mathrm{Re}_{\mathrm{crit}} = \frac{UL}{\nu} = \frac{(r_{\mathrm{crit}}\omega)L}{\nu} \tag{1}$$

$$r_{\mathrm{crit}} = \frac{\nu\,\mathrm{Re}_{\mathrm{crit}}}{\omega L} = \frac{\left(1.5*10^{-5}\ \mathrm{m^2/s}\right)(500{,}000)}{\left(20.94\ \mathrm{rad/s}\right)\left(53*10^{-2}\ \mathrm{m}\right)}$$

$$\text{(Note: } \omega = \left(200\ \mathrm{rot/min}\right)\left(\frac{2\pi\ \mathrm{rad}}{\mathrm{rot}}\right)\left(\frac{\mathrm{min}}{60\ \mathrm{s}}\right) = 20.94\ \mathrm{rad/s}\text{)}$$

$$\boxed{\therefore r_{\mathrm{crit}} = 0.68\ \mathrm{m}} \tag{2}$$

To determine the boundary layer thickness at the blade tip trailing edge, first calculate the Reynolds number there.

$$\mathrm{Re}_{\mathrm{L}} = \frac{UL}{\nu} = \frac{(R\omega)L}{\nu} = \frac{(7.3\ \mathrm{m})\left(20.94\ \mathrm{rad/s}\right)\left(53*10^{-2}\ \mathrm{m}\right)}{\left(1.5*10^{-5}\ \mathrm{m^2/s}\right)} = 5.40*10^{6} \tag{3}$$

$$\therefore \mathrm{Re}_{\mathrm{L}} = 5.40*10^{6} \quad \Rightarrow \text{ the flow is turbulent}$$

Using the turbulent boundary layer correlations:

$$\frac{\delta}{L} = \frac{0.382}{\mathrm{Re}_L^{1/5}} \tag{4}$$

$$\delta = \frac{0.382\left(53*10^{-2}\ \mathrm{m}\right)}{\left(5.40*10^{6}\right)^{1/5}}$$

$$\boxed{\therefore \delta = 9.1*10^{-3}\ \mathrm{m} = 9.1\ \mathrm{mm}} \tag{5}$$

In order to determine the power required to drive the rotor, first determine the torque resulting from the skin friction drag acting on the blades.

$$dF_{\text{1-blade}} = \underbrace{2}_{\substack{\text{two}\\\text{sides}}} C_D \tfrac{1}{2}\rho \underbrace{U}_{=r\omega}{}^2 \underbrace{dA}_{=Ldr} \quad \text{where } C_D = \frac{0.0742}{\mathrm{Re}_L^{1/5}} \quad \text{(turbulent BL correlation)} \tag{6}$$

$$dT_{\text{1-blade}} = r\,dF_{\text{1-blade}} \tag{7}$$

$$\begin{aligned}
T_{\text{4-blades}} &= 4\int_{r=0}^{r=R} dT_{\text{1-blade}} = 4\int_{r=0}^{r=R} r\,dF_{\text{1-blade}}\\
&= 4\int_{r=0}^{r=R} r\,2\frac{0.0742}{\mathrm{Re}_L^{1/5}}\tfrac{1}{2}\rho\left(r\omega\right)^2 L\,dr\\
&= 4\int_{r=0}^{r=R} r\frac{0.0742}{\left(\frac{r\omega L}{\nu}\right)^{1/5}}\rho\left(r\omega\right)^2 L\,dr\\
&= 2.97*10^{-1}\rho\nu^{1/5}\omega^{9/5}L^{4/5}\int_{r=0}^{r=R} r^{14/5}dr
\end{aligned}$$

$$\therefore T_{\text{4-blades}} = 7.81*10^{-2}\rho\nu^{1/5}\omega^{9/5}L^{4/5}R^{19/5} \tag{8}$$

$$\boxed{P_{\text{4-blades}} = \omega T_{\text{4-blades}} = 7.81*10^{-2}\rho\nu^{1/5}\omega^{14/5}L^{4/5}R^{19/5}} \tag{9}$$

For:

$\rho = 1.2\ \text{kg/m}^3$

$\nu = 1.5*10^{-5}\ \text{m}^2/\text{s}$

$\omega = 20.94\ \text{rad/s}$

$L = 53*10^{-2}\ \text{m}$

$R = 7.3\ \text{m}$

$\Rightarrow \quad T_{\text{4-blades}} = 2790\ \text{N}\cdot\text{m} \Rightarrow \boxed{P_{\text{4-blades}} = 58.3\ \text{kW}}$