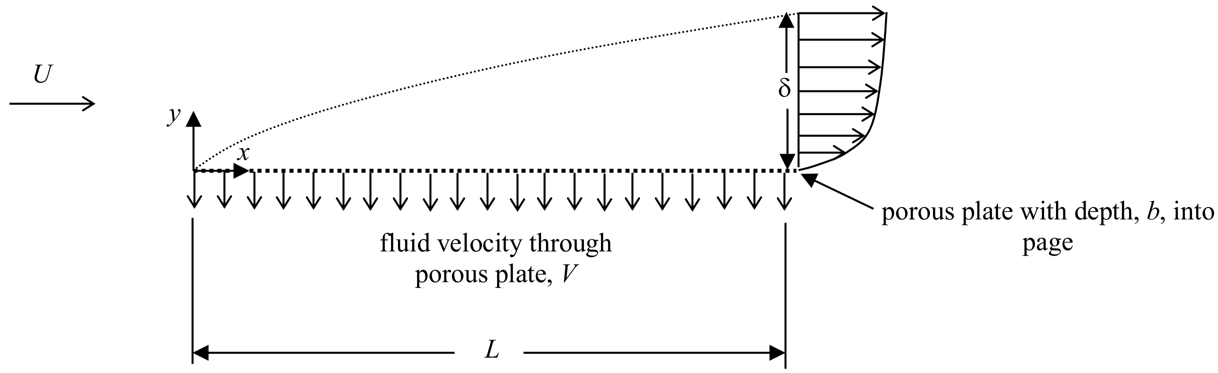
One method proposed to decrease drag and avoid boundary layer separation on aircraft is to use suction to remove the low momentum fluid near the aircraft surface. By removing the low momentum fluid near the surface, the boundary layer remains more stable and transition to a turbulent boundary layer is delayed. This method has been attempted in practice (*Aviation Week & Space Technology*, Oct. 12, 1998, pg. 42). Airbus tested a micro-perforated titanium skin on an Airbus A320 aircraft fin. The ultimate goal of Airbus' tests was to reduce wing drag by 10-16% and empenage/nacelles drag by nearly 5%. Fuel consumption was expected to decrease by as much as 13%.

To analyze this flow, consider a laminar boundary layer on a porous flat plate. Fluid is removed through the plate at a uniform velocity, V. The thickness of the boundary layer is denoted by δ and the velocity outside the boundary layer is a constant, U. Assuming that the velocity profile, u, is given by a power law expression (n is a positive constant describing the shape of the profile and y is the vertical distance from the surface of the plate):

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

Determine:
1. the momentum thickness of the boundary layer in terms of δ
2. the drag acting on the plate over a length L if the plate has a depth b into the page (express your answer in terms of δ_M.)



SOLUTION:

Determine the momentum thickness from its definition,

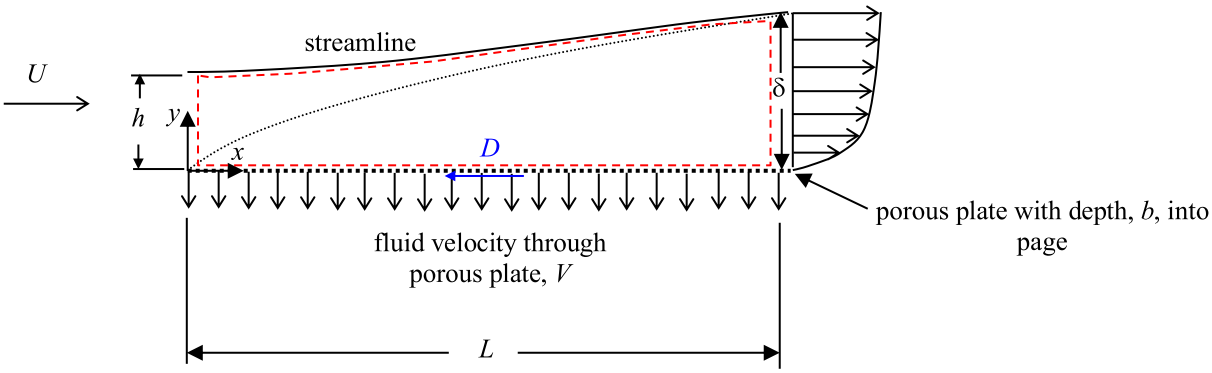
$$\delta_M = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$= \int_0^\delta \left(\frac{y}{\delta}\right)^{1/n}\left[1-\left(\frac{y}{\delta}\right)^{1/n}\right]dy = \delta\int_0^1 \left(\frac{y}{\delta}\right)^{1/n}\left[1-\left(\frac{y}{\delta}\right)^{1/n}\right]d\left(\frac{y}{\delta}\right) = \delta\int_0^1\left[\left(\frac{y}{\delta}\right)^{1/n}-\left(\frac{y}{\delta}\right)^{2/n}\right]d\left(\frac{y}{\delta}\right)$$

$$= \delta\left[\frac{1}{\frac{1}{n}+1}-\frac{1}{\frac{2}{n}+1}\right] = \delta\left[\frac{n}{n+1}-\frac{n}{n+2}\right] = \delta\left[\frac{n(n+2)-n(n+1)}{(n+1)(n+2)}\right]$$

$$\boxed{\therefore \delta_M = \frac{n}{(n+1)(n+2)}\delta} \quad (1)$$

Determine the drag using the linear momentum equation in the *x*-direction using the control volume shown below.



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x\left(\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x}$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{CS} u_x\left(\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \rho U^2 b \int_{y=0}^{y=\delta}\left(\frac{y}{\delta}\right)^{2/n} dy - \rho U^2 bh$$

$$= \rho U^2 b\delta \int_{y/\delta=0}^{y/\delta=1}\left(\frac{y}{\delta}\right)^{2/n} d\left(\frac{y}{\delta}\right) - \rho U^2 bh$$

$$= \frac{\rho U^2 b\delta}{\frac{2}{n}+1} - \rho U^2 bh = \rho U^2 b\delta\left(\frac{n}{n+2}-\frac{h}{\delta}\right)$$

$$F_{B,x} = 0$$

$F_{S,x} = -D$ (Since U is a constant, the pressure will remain constant.)

Substituting and simplifying gives,

$$D = \rho U^2 b\delta\left(\frac{h}{\delta} - \frac{n}{n+2}\right) \tag{2}$$

The height, *h*, can be found using conservation of mass on the same control volume,

$$\frac{d}{dt}\int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0$$

where,

$$\frac{d}{dt}\int_{\text{CV}} \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{\text{CS}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = \rho U b \int_{y=0}^{y=\delta} \left(\frac{y}{\delta}\right)^{1/n} dy - \rho U b h + \rho V b L = \rho U b \delta \int_{y/\delta=0}^{y/\delta=1} \left(\frac{y}{\delta}\right)^{1/n} d\left(\frac{y}{\delta}\right) - \rho U b h + \rho V b L$$

$$= \rho U b \delta\left(\frac{1}{\frac{1}{n}+1}\right) - \rho U b h + \rho V b L = \rho U b \delta\left(\frac{n}{n+1} - \frac{h}{\delta} + \frac{V}{U}\frac{L}{\delta}\right)$$

Substituting and simplifying gives,

$$\rho U b \delta\left(\frac{n}{n+1} - \frac{h}{\delta} + \frac{V}{U}\frac{L}{\delta}\right) = 0$$

$$\frac{h}{\delta} = \frac{n}{n+1} + \frac{V}{U}\frac{L}{\delta} \tag{3}$$

Substituting Eqn. (3) into Eqn. (2) gives,

$$\boxed{D = \rho U^2 b\delta\left[\frac{n}{(n+1)(n+2)} + \frac{V}{U}\frac{L}{\delta}\right]} \tag{4}$$

or

$$\boxed{D = \rho U^2 b\left(\delta_M + \frac{V}{U}L\right)} \tag{5}$$