4.7.2. Heat Transfer and Work in Internally Reversible, State State, Steady Flow Processes

From the First Law, assuming one inlet and one outlet, steady state, and steady flow,

$$\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = -\frac{\dot{Q}_{\text{into CV}}}{\dot{m}} + (h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1). \tag{4.146}$$

Making use of the Tds equation to re-write the specific enthalpy term,

$$Tds = dh - vdp \implies \int_1^2 Tds = h_2 - h_1 - \int_1^2 vdp \implies h_2 - h_1 = \int_1^2 Tds + \int_1^2 vdp. \tag{4.147}$$

Substitute Eq. (4.147) into Eq. (4.146) and simplify to get,

$$\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = -\frac{\dot{Q}_{\text{into CV}}}{\dot{m}} + \int_1^2 Tds + \int_1^2 vdp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1). \tag{4.148}$$

If we further assume that *the process is internally reversible*, then,

$$d\dot{S} = \left.\frac{\delta\dot{Q}_{\text{into CV}}}{T}\right|_{\text{int. rev.}} \implies \dot{m}ds = \left.\frac{\delta\dot{Q}_{\text{into CV}}}{T}\right|_{\text{int. rev.}} \implies \frac{\dot{Q}_{\text{into CV,int. rev.}}}{\dot{m}} = \int_1^2 Tds. \tag{4.149}$$

Substituting into Eq. (4.148),

$$\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = -\frac{\dot{Q}_{\text{into CV,int. rev.}}}{\dot{m}} + \left(\frac{\dot{Q}_{\text{into CV,int. rev.}}}{\dot{m}}\right) + \int_1^2 vdp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1), \tag{4.150}$$

$$\boxed{\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = \int_1^2 vdp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)}, \tag{4.151}$$

or, alternatively,

$$\boxed{\frac{\dot{W}_{\text{other,by CV}}}{\dot{m}} = -\int_1^2 vdp + \frac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)}. \tag{4.152}$$

This is the First Law for an internally reversible, steady state, steady flow with one inlet and one outlet.

Notes:

(1) For an isothermal process ($T =$ constant), we can integrate Eq. (4.149) to get,

$$\dot{m}(s_2 - s_1) = \frac{\dot{Q}_{\text{into CV}}\big|_{\text{int. rev.}}}{T} \implies \frac{\dot{Q}_{\text{into CV}}\big|_{\text{int. rev.}}}{\dot{m}} = T(s_2 - s_1). \tag{4.153}$$

(2) For the case where there is no "other" work, e.g., there is no shaft or electrical work, Eq. (4.152) becomes,

$$\boxed{\int_1^2 vdp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0}, \tag{4.154}$$

which is known as Bernoulli's Equation. Bernoulli's equation is arguably the most frequently used relation in fluid mechanics. It's also frequently used incorrectly since the assumptions (steady state, steady flow, one inlet and one outlet, internally reversible, and no "other" work) must be satisfied. For an incompressible fluid, $v =$ constant and Bernoulli's equation becomes,

$$\boxed{v(p_2 - p_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0}. \tag{4.155}$$

Recall that $v = 1/\rho$ so the previous equation may also be written as,

$$\boxed{\frac{p_2 - p_1}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0}. \tag{4.156}$$

For an ideal gas,

$$v = \frac{RT}{p} \implies \int_1^2 \frac{RT}{p} dp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0. \tag{4.157}$$

For an isothermal process involving an ideal gas,

$$RT \ln\left(\frac{p_2}{p_1}\right) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0. \tag{4.158}$$

For an isentropic process involving an ideal gas,

$$0 = c_p \frac{dT}{T} - R\frac{dp}{p} \implies \frac{dp}{p} = \frac{c_p}{R}\frac{dT}{T}, \tag{4.159}$$

$$\implies \int_1^2 RT\frac{c_p}{R}\frac{dT}{T} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0, \tag{4.160}$$

$$\therefore \underbrace{\int_1^2 c_p dT}_{=h(T_2)-h(T_1)} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0. \tag{4.161}$$

If isentropic flow of a perfect gas is considered (c_p = constant), then the previous equation becomes,

$$c_p(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0, \tag{4.162}$$

$$(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0, \tag{4.163}$$

$$\boxed{\Delta h_T = 0}. \tag{4.164}$$

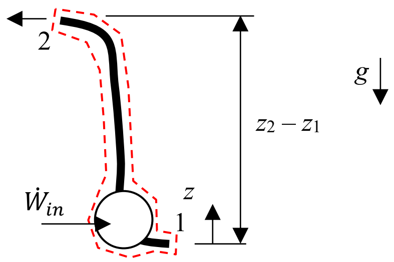
Note that when gases are considered, the potential energy changes are usually very small when compared to the other terms in Bernoulli's equation and can be neglected.

Bernoulli's equation (in the forms given here; there are other forms in which some of the assumptions are relaxed) can be viewed as a statement of the First Law with the assumptions of steady state and steady flow, one inlet and one outlet, internally reversible flow, and a flow with no "other" work.

A 3 hp pump operating at steady state draws in liquid water at 1 atm (abs), 60 °F and delivers it at 5 atm (abs) at an elevation 20 ft above the inlet. There is no significant change in velocity between the inlet and exit. Is it possible to pump 1000 gal in 10 min or less? Explain.

Image: https://www.bobvila.com/articles/some-advice-about-sump-pumps/

SOLUTION:



The mass flow rate required to pump 1000 gal of liquid water in 10 min is,

$$\dot{m} = \rho Q \,, \tag{1}$$

where ρ is the density of liquid water, assumed here to be 62.4 lb_m/ft^3, and Q is the volumetric flow rate,

$$Q = (1000 \text{ gal})/(10 \text{ min}) = 100 \text{ gal/min} = 13.37 \text{ ft}^3\text{/min} = 0.223 \text{ ft}^3\text{/s} \tag{2}$$

=> $\dot{m}$ = 13.9 lb_m/s.

Since we're interested in knowing if the pump is capable of pumping at the given flow rate, consider the ideal case, i.e., assume internally reversible, adiabatic flow. Note that the flow is at steady state and has one inlet and outlet. For these conditions, the 1st Law may be written as,

$$\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = \int_{p_1}^{p_2} v\,dp + \frac{1}{2}\left(V_2^2 - V_1^2\right) + g\left(z_2 - z_1\right). \tag{3}$$

For the current situation, assume the liquid water is incompressible (also re-write the specific volume as $v = 1/\rho$). Furthermore, we're told that there's no significant change in the velocity between the inlet and outlet, so the change in kinetic energy may be neglected. Re-writing Eq. (3) for these conditions gives,

$$\frac{\dot{W}_{\text{other,on CV}}}{\dot{m}} = \frac{p_2 - p_1}{\rho} + g\left(z_2 - z_1\right), \tag{4}$$

$$\dot{m} = \frac{\dot{W}_{\text{other,on CV}}}{\dfrac{p_2 - p_1}{\rho} + g\left(z_2 - z_1\right)}. \tag{5}$$

Using the given parameters,

$\dot{W}_{\text{other,on CV}}$ = 3 hp = 1650 ft.lb_f/s,

p_1 = 1 atm (abs) = 2117 lb_f/ft^2, p_2 = 5 atm (abs) = 10580 lb_f/in^2,

ρ = 62.4 lb_m/ft^3,

g = 32.2 ft/s^2, $z_2 - z_1$ = 20 ft,

=> $\dot{m}$ = 10.6 lb_m/s.

Since the ideal mass flow rate is smaller than what is required, it's not possible to pump the water at the desired flow rate.