4.7. The Second Law of Thermodynamics for a Control Volume

As with COM, the LME, the AME, and the First Law, we will convert our system form of the Second Law to a control volume form using the Reynolds Transport Theorem. To do so, we should first write the Second Law in terms of time rates of change (refer to Eq. (3.149)),

$$\underbrace{\frac{DS}{Dt}}_{\substack{\text{rate of increase}\\ \text{of entropy in system}}} = \underbrace{\int_b \frac{\delta \dot{Q}_{\text{into sys}}}{T}}_{\substack{\text{rate at which entropy}\\ \text{enters the system via}\\ \text{heat transfer through}\\ \text{the boundary}}} + \underbrace{\dot{\sigma}}_{\substack{\text{rate at which}\\ \text{entropy is produced}\\ \text{in the system due to}\\ \text{internal irreversibilities}}}, \tag{4.137}$$

where the Lagrangian derivative notation has been used to remind us that we're following a system. Recall that $\dot{\sigma} \geq 0$ with the equality holding only for internally reversible processes. Now let's write the left-hand side of the equation in terms of an integral so that we can have variations in the system specific entropy,

$$\frac{D}{Dt}\int_{V_{\text{sys}}} s\rho dV = \int_b \frac{\delta \dot{Q}_{\text{into sys}}}{T} + \dot{\sigma}. \tag{4.138}$$

where s is the specific entropy. After applying the Reynolds Transport Theorem to convert to a control volume perspective,

$$\boxed{\underbrace{\frac{d}{dt}\int_{CV} s\rho dV}_{\substack{\text{rate of entropy}\\ \text{increase in CV}}} + \underbrace{\int_{CS} s\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}_{\substack{\text{net rate at which entropy}\\ \text{leaves the CV through the CS}}} = \underbrace{\int_{CS} \frac{\delta \dot{Q}_{\text{into CV}}}{T}}_{\substack{\text{rate at which entropy}\\ \text{enters the CV via heat transfer}}} + \underbrace{\dot{\sigma}}_{\substack{\text{rate at which entropy}\\ \text{is produced in the CV}\\ \text{due to internal irreversibilities}}}} \tag{4.139}$$

Note that the subscript on the heat transfer integral was changed from "b" for "boundary" to "CS" for control surface. This is the control volume form of the Second Law of Thermodynamics (aka the Entropy Equation)!

Notes:

(1) The rate of entropy increase within the control volume (the first term on the left-hand side of Eq. (4.139)) may also be written as,

$$\frac{dS_{CV}}{dt} = \frac{d}{dt}\int_{CV} s\rho dV. \tag{4.140}$$

The net flow rate of entropy *out* of the control volume due to mass flowing out of and into the control volume may be written as,

$$\sum_{\text{all outlets}} s\dot{m} - \sum_{\text{all inlets}} s\dot{m} = \int_{CS} s\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right). \tag{4.141}$$

Combined together, the Second Law for a control volume may then be written as,

$$\frac{dS_{CV}}{dt} = \sum_{\text{all inlets}} \dot{m}s - \sum_{\text{all outlets}} \dot{m}s + \int_{CS} \frac{\delta \dot{Q}_{\text{into CV}}}{T} + \dot{\sigma}. \tag{4.142}$$

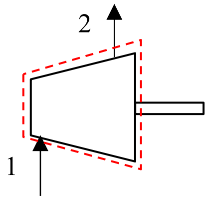
(2) Recall that entropy production is related to the degree of irreversibility in a system/control volume. The larger the change in entropy, the further the system/control volume is from being reversible or ideal. Hence, Eq. (4.139) can be used to determine situations that result in inefficiencies.

Let's consider some examples to see how the Second Law is applied to CVs.

Steam enters a turbine operating at steady state at 1 MPa (abs) and 200 °C and exits at 40 °C with a quality of 83%. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine:

a. the power developed by the turbine per unit mass of steam, and
b. the change in the specific entropy from the inlet to the exit per unit mass of steam.

SOLUTION:
To determine the power developed by the turbine, apply the 1st Law to a control volume surrounding the turbine.



$$\frac{dE_{\text{CV}}}{dt}+\sum_{\text{out}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m}-\sum_{\text{in}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m}=\dot{Q}_{\text{into}}+\dot{W}_{\text{on,other}}\,, \tag{1}$$

where,

$dE_{\text{CV}}/dt = 0$ (steady flow)

$\Delta(^1/_2V^2)$ and $\Delta(gz)$ are assumed to be negligibly small compared to the specific enthalpy

$\dot{Q}_{\text{into}} = 0$ (adiabatic flow)

$\dot{m}_{\text{out}} = \dot{m}_{\text{in}} = \dot{m}$ (from conservation of mass)

$$\Rightarrow \frac{\dot{W}_{\text{on,other}}}{\dot{m}} = h_2 - h_1\,. \tag{2}$$

The specific enthalpies are,

$h_1 = 2827.9$ kJ/kg (@ 200 °C and 1 MPa = 10 bar ⇒ superheated vapor; from Table A-4 in Moran et al., 7th ed.)

$$h_2 = x_2h_v + (1 - x_2)h_l \tag{3}$$

$x_2 = 0.83$, $h_v = 2574.3$ kJ/kg, $h_l = 167.57$ kJ/kg
(@ 40 °C, two-phase, liquid-vapor state; from Table A-2 in Moran et al., 7th ed.)

$\Rightarrow h_2 = 2165.1$ kJ/kg.

$\Rightarrow \boxed{\frac{\dot{W}_{\text{on,other}}}{\dot{m}} = \text{-662.8 kJ/kg}}$ (work is being done by the system).

The rate of entropy production per unit mass of steam may be found using,

$$\frac{dS_{CV}}{dt}=\sum_{\text{in}}s\dot{m}-\sum_{\text{out}}s\dot{m}+\int_b\frac{\dot{Q}_{\text{into}}}{T}+\dot{\sigma}\Rightarrow\dot{\sigma}=\frac{dS_{CV}}{dt}+\sum_{\text{out}}s\dot{m}-\sum_{\text{in}}s\dot{m}-\int_b\frac{\dot{Q}_{\text{into}}}{T}\,, \tag{4}$$

where,

$$\frac{dS_{CV}}{dt}=0 \quad \text{(steady flow assumed)}, \tag{5}$$

$$\sum_{\text{out}}s\dot{m}-\sum_{\text{in}}s\dot{m}=\dot{m}\left(s_2-s_1\right) \quad \text{(mass flow rate constant from Cons. of Mass)}, \tag{6}$$

$$\int_b\frac{\dot{Q}_{\text{into}}}{T}=0 \quad \text{(adiabatic process)}, \tag{7}$$

$$\Rightarrow \quad \dot{\sigma}=\dot{m}\left(s_2-s_1\right)\Rightarrow\frac{\dot{\sigma}}{\dot{m}}=s_2-s_1\,. \tag{8}$$

The change in specific entropy is,

$s_1 = 6.6940$ kJ/(kg.K) (@ 200 °C and 1 MPa = 10 bar ⇒ superheated vapor; from Table A-4 in Moran et al., 7th ed.)

$$s_2 = x_2 s_v + (1 - x_2) s_l \tag{9}$$

$x_2 = 0.83$, $s_v = 8.2570$ kJ/kg, $s_l = 0.5725$ kJ/kg

(@ 40 °C, two-phase, liquid-vapor state; from Table A-2 in Moran et al., 7th ed.)

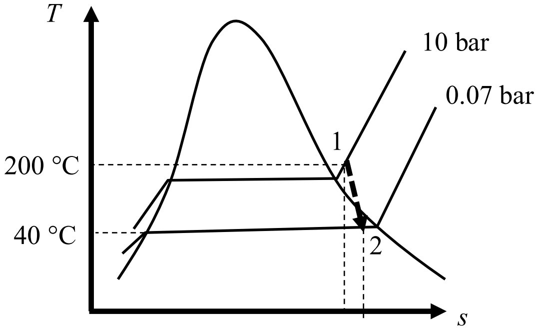
$\Rightarrow s_2 = 6.9506$ kJ/(kg.K).

$\Rightarrow s_2 - s_1 = 0.2566$ kJ/(kg.K).

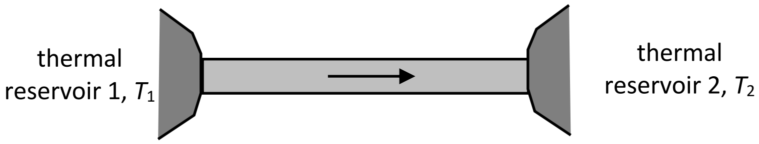
$\Rightarrow \boxed{\dot{\sigma}/\dot{m} = 0.2566 \text{ kJ/(kg.K)}}$.

Note that the process is not internally reversible since $\dot{\sigma} > 0$.

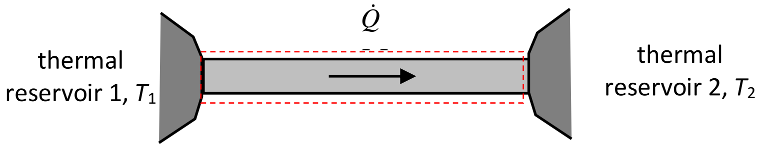
A plot of the process on a *T*-*s* diagram is shown in the following figure.



Consider a steel rod steadily conducting heat from thermal reservoir 1 into thermal reservoir 2. Using the entropy rate balance equation, show that $T_1 > T_2$. Note that T_1 and T_2 are absolute temperatures.



SOLUTION:



Apply the entropy rate balance equation to a control volume surrounding the steel rod,

$$\frac{dS_{CV}}{dt} = \sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} + \int_b \frac{\delta \dot{Q}_{\text{into CV}}}{T} + \dot{\sigma} \ , \tag{1}$$

where,

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(steady)}, \tag{2}$$

$$\sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} = 0 \quad \text{(no flow in or out of the CV)}, \tag{3}$$

$$\int_b \frac{\delta \dot{Q}_{\text{into CV}}}{T} = \frac{\dot{Q}}{T_1} - \frac{\dot{Q}}{T_2} \ , \tag{4}$$

where, from the 1st Law applied to the same CV, the rate of heat transfer through the rod remains constant. In addition, the temperature at the left boundary, where the heat enters the CV, i.e., $\dot{Q}_{\text{into CV,1}} = \dot{Q}$, is T_1 and the temperature at the right boundary, where heat leaves the CV, i.e., $\dot{Q}_{\text{into CV,2}} = -\dot{Q}$, is T_2.

Substitute and simplify,

$$0 = \frac{\dot{Q}}{T_1} - \frac{\dot{Q}}{T_2} + \dot{\sigma} \ , \tag{5}$$

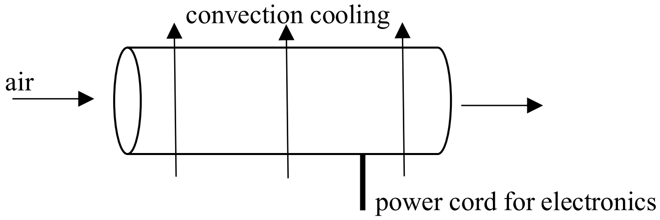
$$\dot{\sigma} = \dot{Q}\left(\frac{T_1 - T_2}{T_1 T_2}\right). \tag{6}$$

Since $\dot{\sigma} \geq 0$, $T_1 \geq T_2$. The equality, $T_1 = T_2$, results in an internally reversible process whereas a finite temperature difference, $T_1 > T_2$, results in an internally irreversible process.

This result also demonstrates the equivalence between entropy generation via heat transfer and the Clausius statement of the 2nd Law.

Electronic components are mounted on the inner surface of a horizontal cylindrical duct with an inner diameter of 0.2 m. To prevent overheating of the electronics, the cylinder is cooled by a stream of air flowing through the cylinder and by convection from its outer surface. Air enters the duct at 25 °C, 1 bar (abs), and a speed of 0.3 m/s, and exits at 40 °C with negligible changes in kinetic energy and pressure. Convective cooling occurs on the cylinder's outer surface to the surroundings, which are at 25 °C, in accord with hA = 3.4 W/K, where h is the heat transfer coefficient and A is the cylinder's surface area. The electronic components require 0.20 kW of electric power. For steady state conditions, determine:

a. the mass flow rate of air through the cylinder, in kg/s,
b. the temperature on the outer surface of the duct, and
c. the rate of entropy production in the air passing through the duct.



SOLUTION:
The mass flow rate through the cylinder is,

$$\dot{m} = \rho_1 V_1 A_1 = \left(\frac{p_1}{RT_1}\right) V_1 \left(\frac{\pi D_1^2}{4}\right), \tag{1}$$

where the ideal gas law has been used. Using the given data,

p_1 = 1 bar (abs) = 1*10^5 Pa
R_{air} = 287 J/(kg.K)
T_1 = 25 °C = 298 K
V_1 = 0.3 m/s
D_1 = 0.2 m
=> $\boxed{\dot{m} = 0.011 \text{ kg/s}}$.

Now apply the 1st Law the control volume shown in the figure,

$$\frac{dE_{CV}}{dt} = \sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\text{into,CV}} + \dot{W}_{\text{other,on CV}}, \tag{2}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(steady conditions)}, \tag{3}$$

$$\sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \dot{m}\left(h_1 - h_2\right) = \dot{m}c_p\left(T_1 - T_2\right), \tag{4}$$

(assuming changes in kinetic energy and potential energy are negligible; the mass flow at the inlet and outlet are the same from conservation of mass; the air behaves as a perfect gas),

$$\dot{Q}_{\text{into,CV}} = -hA\left(T_{\text{surface}} - T_{\text{surr}}\right) \quad \text{(Newton's Law of Convection)}, \tag{5}$$

$$\dot{W}_{\text{other,on CV}} = 0.20 \text{ kW} \quad \text{(given power for electronics)}. \tag{6}$$

Substitute and simplify,

$$0 = \dot{m}c_p\left(T_1 - T_2\right) - hA\left(T_{\text{surface}} - T_{\text{surr}}\right) + \dot{W}_{\text{other,on CV}}\,, \tag{7}$$

$$T_{\text{surface}} = T_{\text{surr}} + \frac{\dot{m}c_p\left(T_1 - T_2\right) + \dot{W}_{\text{other,on CV}}}{hA}\,,$$

where,

T_{surr} = 25 °C = 298 K (given),
$\dot{m}$ = 0.011 kg/s (found previously),
c_p = 1.005 kJ/(kg.K) (Table A-20 in Moran et al., 8th ed.)
T_1 = 25 °C = 298 K (given),
T_2 = 40 °C = 313 K (given),
$\dot{W}_{\text{other,on CV}} = 0.20$ kW (given),
hA = 3.4 W/K (given),
=> $\boxed{T_{\text{surface}} = 308 \text{ K} = 35.1\ ^\circ\text{C}}$.

The rate at which entropy is produced in the air passing through the duct is found by applying the entropy equation to the same control volume,

$$\frac{dS_{CV}}{dt} = \sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} + \int_b \frac{\dot{Q}_{\text{into}}}{T} + \dot{\sigma}\,, \tag{8}$$

where

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(steady)}, \tag{9}$$

$$\sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} = \dot{m}\left(s_1 - s_2\right) = \dot{m}\left[c_p \ln\left(\frac{T_1}{T_2}\right) - R\ln\left(\frac{p_1}{p_2}\right)\right], \tag{10}$$

(where a perfect gas assumption has been used to determine the change in specific entropy),

$$\int_b \frac{\dot{Q}_{\text{into}}}{T} = \frac{-hA\left(T_{\text{surface}} - T_{\text{surr}}\right)}{T_{\text{surface}}}\,, \tag{11}$$

(Note that the absolute temperature in the denominator is the temperature where the heat transfer occurs on the control surface boundary.)

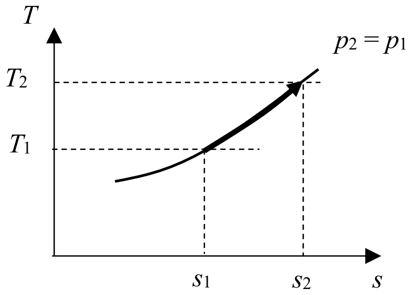
Substitute and simplify,

$$0 = \dot{m}\left[c_p \ln\left(\frac{T_1}{T_2}\right) - R\ln\left(\frac{p_1}{p_2}\right)\right] + \frac{-hA\left(T_{\text{surface}} - T_{\text{surr}}\right)}{T_{\text{surface}}} + \dot{\sigma}\,, \tag{12}$$

$$\dot{\sigma} = \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right] + \frac{hA\left(T_{\text{surface}} - T_{\text{surr}}\right)}{T_{\text{surface}}}\,. \tag{13}$$

Using the given data,

T_{surr} = 25 °C = 298 K (given),
$\dot{m}$ = 0.011 kg/s (found previously),
c_p = 1.005 kJ/(kg.K) (Table A-20 in Moran et al., 8th ed.)
T_1 = 25 °C = 298 K (given),
T_2 = 40 °C = 313 K (given),
$p_2 = p_1$ (given),
T_{surface} = 308 K (found previously),
$\dot{W}_{\text{other,on CV}} = 0.20$ kW (given),
hA = 3.4 W/K (given),
=> $\boxed{\dot{\sigma} = 0.653 \text{ W/K}}$.



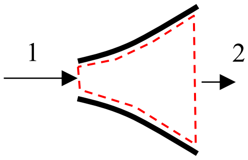
Air enters a diffuser operating at steady state at 4 bar (abs) and 290 K with a speed of 512 m/s. The exit speed is 110 m/s. For adiabatic operation with no internal irreversibilities, determine:

a. the exit temperature, in K, and
b. the exit pressure, in bar (abs).

SOLUTION:

Apply the 1st Law to the control volume shown in the figure,

$$\frac{dE_{CV}}{dt} = \sum_{\text{in}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} - \sum_{\text{out}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} + \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}, \tag{1}$$



where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (steady)}, \tag{2}$$

$$\sum_{\text{in}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} - \sum_{\text{out}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} = \dot{m}\left[\left(h_1+\tfrac{1}{2}V_1^2\right)-\left(h_2+\tfrac{1}{2}V_2^2\right)\right], \tag{3}$$

(Changes in potential energy are assumed negligible since we're dealing with a gas.)

$$\dot{Q}_{\text{into CV}} \text{ (adiabatic process)}, \tag{4}$$

$$\dot{W}_{\text{other,on CV}} = 0 \text{ (no “other” work)}, \tag{5}$$

$$\Rightarrow\; h_2 = h_1 + \tfrac{1}{2}\left(V_1^2 - V_2^2\right). \tag{6}$$

Treating air as an ideal gas,

$h_1(T_1 = 290\text{ K}) = 290.16$ kJ/kg (from Table A-22 in Moran et al., 8th ed.),
$V_1 = 512$ m/s,
$V_2 = 110$ m/s,
=> $h_2 = 415.18$ kJ/kg => $\boxed{T_2 = 414\text{ K}}$ (interpolating in Table A-22). (7)

The exit pressure may be found using the change in entropy relation for an ideal gas,

$$s_2 - s_1 = s_2^0(T_2) - s_1^0(T_1) - R\ln\left(\frac{p_2}{p_1}\right), \tag{8}$$

where,

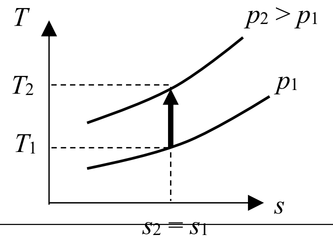
$s_2 - s_1 = 0$ since the process is adiabatic and internally reversible => isentropic (given),
$s^0_2(T_2 = 414\text{ K}) = 2.02676$ kJ/(kg.K) (interpolating in Table A-22),
$s^0_1(T_1 = 290\text{ K}) = 1.66802$ kJ/(kg.K) (Table A-22),
$R_{\text{air}} = 0.287$ kJ/(kg.K),
$p_1 = 4$ bar (abs),
=> $\boxed{p_2 = 14.0\text{ bar (abs)}}$.

If we assumed the air behaves as a perfect gas with $c_p = 1.005$ kJ/(kg.K) (Table A-20 in Moran et al., 8th ed.), then Eq. (6) may be written as,

$$T_2 = T_1 + \tfrac{1}{2c_p}\left(V_1^2 - V_2^2\right) \;\Rightarrow\; T_2 = 414\text{ K (Same answer as before!)} \tag{9}$$

In addition, for a perfect gas undergoing an isentropic process,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \;\Rightarrow\; p_2 = 13.9\text{ bar (abs) (within 1\% of the answer found previously)} \tag{10}$$



Note that if the flow had internal irreversibilities, then the entropy equation applied to the control volume gives,

$$\frac{dS_{CV}}{dt} = \sum_{\text{in}} (s\dot{m}) - \sum_{\text{out}} (s\dot{m}) + \int_b \frac{\dot{Q}_{\text{into}}}{T} + \dot{\sigma} \, , \tag{11}$$

where,

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(steady flow)}, \tag{12}$$

$$\int_b \frac{\dot{Q}_{\text{into}}}{T} = 0 \quad \text{(adiabatic flow)}, \tag{13}$$

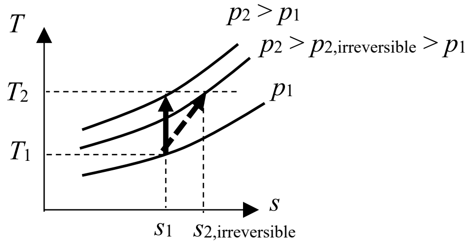
$$\Rightarrow \quad s_2 - s_1 = \frac{\dot{\sigma}}{\dot{m}} \quad \text{where } \dot{\sigma}/\dot{m} > 0 \text{ if the flow is internally irreversible (= 0 if internally reversible)} \tag{14}$$

Substituting into Eq. (8) and solving for p_2 gives,

$$\frac{\dot{\sigma}}{\dot{m}} = s_2^0(T_2) - s_1^0(T_1) - R\ln\left(\frac{p_2}{p_1}\right), \tag{15}$$

$$p_2 = p_1 \exp\left[\frac{s_2^0(T_2) - s_1^0(T_1) - \dot{\sigma}/\dot{m}}{R}\right]. \tag{16}$$

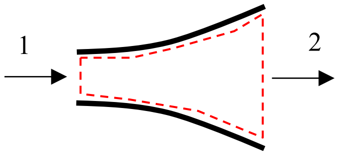
Thus, with internal irreversibilities, the exit pressure is smaller than if the flow was internally reversible.



Nitrogen (N_2) enters a well-insulated diffuser operating at steady state at 0.656 bar (abs), 300 K with a velocity of 282 m/s. The inlet area is $4.8*10^{-3}$ m^2. At the diffuser exit, the pressure is 0.9 bar (abs) and the velocity is 130 m/s. The nitrogen behaves as an ideal gas with a specific heat ratio of 1.4. Determine:

a. the exit temperature, in K,
b. the exit area, in m^2, and
c. the rate of entropy production, in kJ/K per kg of flowing nitrogen.

SOLUTION:



Apply the 1st Law to a control volume surrounding the interior of the diffuser,

$$\frac{dE_{CV}}{dt} = \sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\text{into,CV}} + \dot{W}_{\text{other,on CV}}, \tag{1}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady flow)}, \tag{2}$$

$$\sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \dot{m}\left[\left(h_2 - h_1\right) + \tfrac{1}{2}\left(V_2^2 - V_1^2\right)\right], \tag{3}$$

(changes in PE are assumed negligible compared to changes in the other terms, especially since we're dealing with a gas; from conservation of mass, $\dot{m} = \dot{m}_2 = \dot{m}_1$)

$$\dot{Q}_{\text{into,CV}} = 0 \quad \text{(the diffuser is well-insulated)} \tag{4}$$

$$\dot{W}_{\text{other, on CV}} = 0 \quad \text{(no other work acting on the control volume)} \tag{5}$$

Substituting and simplifying,

$$h_2 = h_1 + \tfrac{1}{2}\left(V_1^2 - V_2^2\right). \tag{6}$$

Assuming ideal gas behavior,

$\bar{h}_1$ (T_1 = 300 K) = 8723 kJ/kmol (Table A-23 in Moran et al., 8th ed.)

$$\Rightarrow \quad h_1 = \bar{h}_1 / M \quad \Rightarrow \quad h_1 = 311.42 \text{ kJ/kg}, \tag{7}$$

where the molecular weight of nitrogen gas (N_2) is 28.01 kg/kmol has been used.

Given V_1 = 282 m/s, V_2 = 130 m/s, and making use of Eqs. (7) and (6),

$$h_2 = 342.73 \text{ kJ/kg} \Rightarrow \bar{h}_2 = 9600. \text{ kJ/kmol} \Rightarrow \boxed{T_2 = 330 \text{ K}}. \tag{8}$$

Interpolation in Table A-23 was used to determine this outlet temperature.

Alternately, if a perfect gas model is assumed, we can write Eq. (6) as,

$$c_p T_2 = c_p T_1 + \tfrac{1}{2}\left(V_1^2 - V_2^2\right) \Rightarrow T_2 = T_1 + \frac{V_1^2 - V_2^2}{2c_p}, \tag{9}$$

where,

$$c_p = \frac{kR}{k-1} = \frac{k\left(\bar{R}_u / M\right)}{k-1}, \tag{10}$$

=> c_p = 1.0389 kJ/(kg.K) with k = 1.4, $\bar{R}_u$ = 8.314 kJ/(kmol.K), and M = 28.01 kg/kmol

=> T_2 = 330 K, which is the same result found previously

The exit area may be found by applying conservation of mass to the same control volume,

$$\frac{dM_{CV}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_1 \Rightarrow \rho_2 V_2 A_2 = \rho_1 V_1 A_1 \Rightarrow A_2 = A_1\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{V_1}{V_2}\right), \tag{11}$$

Note that the process is assumed to be steady. The density ratio may be found by making use of the ideal gas law,

$$\frac{\rho_1}{\rho_2} = \frac{p_1/RT_1}{p_2/RT_2} = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right). \tag{12}$$

Substituting and simplifying,

$$A_2 = A_1\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right). \tag{13}$$

Using the given data
$A_1 = 4.8*10^{-3}$ m^2,
$p_1 = 0.656$ bar (abs),
$p_2 = 0.9$ bar (abs),
$T_2 = 330$ K (calculated for part (a)),
$T_1 = 300$ K,
$V_1 = 282$ m/s,
$V_2 = 130$ m/s,
=> $\boxed{A_2 = 8.35*10^{-3}\text{ m}^2}$.

The rate of entropy production may be found by applying the entropy equation to the same control volume,

$$\frac{dS_{\text{CV}}}{dt} = \sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} + \int_b \frac{\dot{Q}_{\text{into CV}}}{T} + \dot{\sigma}_{\text{CV}}, \tag{14}$$

where,

$$\frac{dS_{\text{CV}}}{dt} = 0 \quad \text{(assuming steady flow)}, \tag{15}$$

$$\sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} = \dot{m}\left(s_1 - s_2\right), \tag{16}$$

$$\int_b \frac{\dot{Q}_{\text{into CV}}}{T} = 0 \quad \text{(assuming adiabatic operation)}, \tag{17}$$

$$\Rightarrow \quad \dot{\sigma}_{\text{CV}} = \dot{m}\left(s_2 - s_1\right) \quad \Rightarrow \quad \frac{\dot{\sigma}_{\text{CV}}}{\dot{m}} = s_2 - s_1 \tag{18}$$

The change in entropy for an ideal gas is,

$$s_2 - s_1 = \frac{\left[\bar{s}_2^0\left(T_2\right) - \bar{s}_1^0\left(T_1\right) - \bar{R}_u \ln\left(\frac{p_2}{p_1}\right)\right]}{M}, \tag{19}$$

where,

$\bar{s}_2^0\left(T_2 = 330\text{ K}\right) = 194.459$ kJ/(kmol.K), (Table A-23 in Moran et al., 8th ed.)

$\bar{s}_1^0\left(T_1 = 300\text{ K}\right) = 191.682$ kJ/(kmol.K), (Table A-23 in Moran et al., 8th ed.)

$\bar{R}_u = 8.314$ kJ/(kmol.K),
p_2/p_1 = (0.9 bar (abs))/(0.656 bar (abs)) = 1.372,
M = 28.01 kg/kmol,
=> $s_2 - s_1 = 5.28*10^{-3}$ kJ/(kg.K).

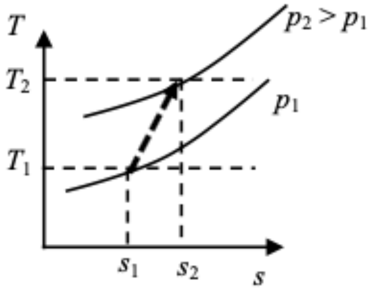
Thus, making use of Eq. (18),

$$\boxed{\frac{\dot{\sigma}_{\text{CV}}}{\dot{m}} = 5.28*10^{-3}\text{ kJ/(kg.K)}}$$

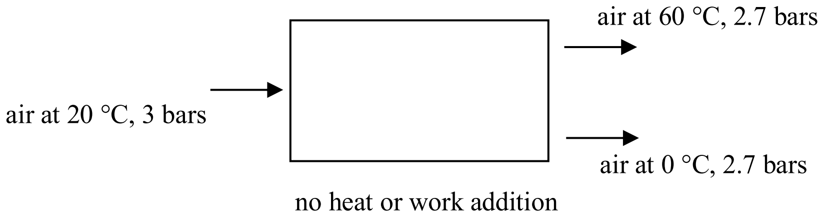
If we assume the nitrogen behaves as an ideal gas, then we could find the change in entropy using,

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right), \tag{20}$$

=> $s_2 - s_1 = 5.15 * 10^{-3}$ kJ/(kg.K), which is within 2.5% of the previous result.

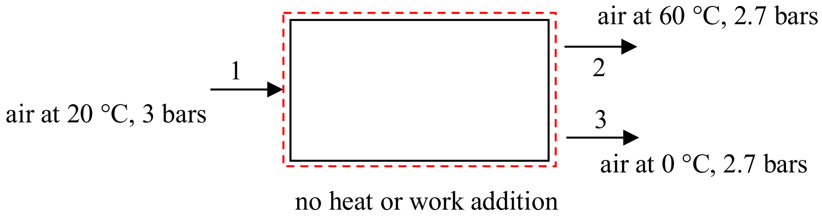


An inventor claims to have developed a device requiring no work input or heat transfer, yet able to produce steady state hot and cold air streams as shown in the figure. Evaluate this claim assuming the ideal gas model for air and ignoring kinetic and potential energy effects.



SOLUTION:

Apply conservation of mass, the First Law, and the Second Law to the control volume shown below.



Conservation of Mass,

$$\frac{d}{dt}\int_{CV}\rho dV+\int_{CS}\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}=0 \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV}\rho dV=0 \quad \text{(steady flow)}$$

$$\int_{CS}\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}=\dot{m}_3+\dot{m}_2-\dot{m}_1$$

Substitute and re-arrange,

$$\dot{m}_3+\dot{m}_2-\dot{m}_1=0$$

$$\frac{\dot{m}_3}{\dot{m}_1}=1-\frac{\dot{m}_2}{\dot{m}_1} \tag{2}$$

First Law of Thermodynamics,

$$\frac{d}{dt}\int_{\mathrm{CV}} e\rho\, dV + \int_{\mathrm{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\mathrm{into}\\ \mathrm{CV}}} + \dot{W}_{\substack{\mathrm{on}\\ \mathrm{CV}}} \tag{3}$$

where,

$$\frac{d}{dt}\int_{\mathrm{CV}} e\rho dV = 0 \quad \text{(steady flow}$$

$$\int_{\mathrm{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{m}_3 h_3 + \dot{m}_2 h_2 - \dot{m}_1 h_1$$

$$\dot{Q}_{\substack{\mathrm{into}\\ \mathrm{CV}}} = \dot{W}_{\substack{\mathrm{on}\\ \mathrm{CV}}} = 0 \quad \text{(no heat or work addition)}$$

Substitute and re-arrange,

$$\dot{m}_3 h_3 + \dot{m}_2 h_2 - \dot{m}_1 h_1 = 0$$

$$\frac{\dot{m}_3}{\dot{m}_1} h_3 = h_1 - \frac{\dot{m}_2}{\dot{m}_1} h_2 \tag{4}$$

Substitute Eq. (2) into Eq. (4) and simplify,

$$\left(1 - \frac{\dot{m}_2}{\dot{m}_1}\right) h_3 = h_1 - \frac{\dot{m}_2}{\dot{m}_1} h_2$$

$$\frac{\dot{m}_2}{\dot{m}_1} = \frac{h_1 - h_3}{h_2 - h_3} \tag{5}$$

From thermodynamics tables for air at the given inlet and outlet temperatures,

h_1 = 293.2 kJ/kg (T_1 = 20 °C = 293 K)
h_2 = 333.3 kJ/kg (T_2 = 60 °C = 333 K)
h_3 = 273.1 kJ/kg (T_3 = 0 °C = 273 K)

Hence,

$$\frac{\dot{m}_2}{\dot{m}_1} = 0.334 \quad \text{and} \quad \frac{\dot{m}_3}{\dot{m}_1} = 0.666 \tag{6}$$

Note that the outgoing mass flow rates are both positive, consistent with the problem description.

Entropy Equation,

$$\frac{d}{dt}\int_{CV} s\rho dV + \int_{CS} s(\rho \boldsymbol{u}_{rel} \cdot d\boldsymbol{A}) = \int_{CS} \frac{\delta \dot{Q}_{into}}{T} + \dot{\sigma}, \tag{7}$$

where,

$$\frac{d}{dt}\int_{\mathrm{CV}} s\rho dV = 0 \quad \text{(steady state)}$$

$$\int_{\mathrm{CS}} s\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{m}_3 s_3 + \dot{m}_2 s_2 - \dot{m}_1 s_1$$

$$\int_{CS} \frac{\delta \dot{Q}_{into}}{T} = 0 \text{ (no heat added to the control volume)}$$

Substitute and re-arrange,

$$\dot{m}_3 s_3 + \dot{m}_2 s_2 - \dot{m}_1 s_1 = \dot{\sigma},$$

$$\frac{\dot{m}_3}{\dot{m}_1} s_3 + \frac{\dot{m}_2}{\dot{m}_1} s_2 - s_1 = \frac{\dot{\sigma}}{\dot{m}_1},$$

$$\left(1 - \frac{\dot{m}_2}{\dot{m}_1}\right) s_3 + \frac{\dot{m}_2}{\dot{m}_1} s_2 - s_1 = \frac{\dot{\sigma}}{\dot{m}_1},$$

$$(s_3 - s_1) + \frac{\dot{m}_2}{\dot{m}_1}(s_2 - s_3) = \frac{\dot{\sigma}}{\dot{m}_1}. \tag{8}$$

Note that for an ideal gas, the specific entropy at a given temperature and pressure can be determined by,

$$s\left(T_B, p_B\right) - s\left(T_A, p_A\right) = s^0\left(T_B\right) - s^0\left(T_A\right) - R\ln\frac{p_B}{p_A} \tag{9}$$

Substituting Eq. (9) into Eq. (8) gives,

$$\left(s_3^0 - s_1^0 - R\ln\frac{p_3}{p_1}\right) + \frac{\dot{m}_2}{\dot{m}_1}\left(s_2^0 - s_3^0 - R\ln\frac{p_2}{p_3}\right) = \frac{\dot{\sigma}}{\dot{m}_1}. \tag{10}$$

From thermodynamics tables for air at the given conditions,

$s^0{}_1$ = 1.6783 kJ/(kg·K) (T_1 = 20 °C = 293 K)
$s^0{}_2$ = 1.8069 kJ/(kg·K) (T_2 = 60 °C = 333 K)
$s^0{}_3$ = 1.6073 kJ/(kg·K) (T_3 = 0 °C = 273 K)

and from the given conditions,

p_1 = 3.0 bar
p_2 = 2.7 bar
p_3 = 2.7 bar

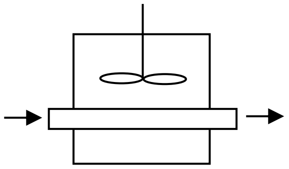
Thus, Eq. (10) simplifies to,

$$\frac{\dot{\sigma}}{\dot{m}_1} = 0.0259 \text{ kJ/(kg.K)}. \tag{11}$$

Thus, the Second Law of Thermodynamics is satisfied.

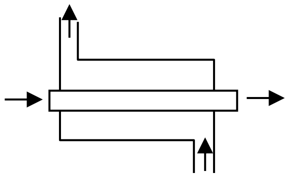
Since conservation of mass, the First Law, and the Second Law are all satisfied, the claim of the inventor is not unreasonable.

Two alternate systems are under consideration for bringing a stream of air from 17 °C to 52 °C at an essentially constant pressure of 1 bar.



Air temperature increases as a consequence of the stirring of a liquid surrounding the line carrying the air.

METHOD 1



Air temperature increases by passing it through one side of a counterflow heat exchanger. On the other side, steam condenses at a pressure of 1 bar from saturated vapor to saturated liquid.

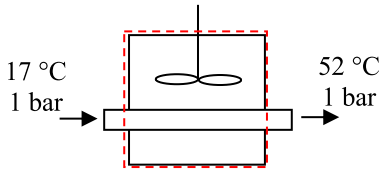
METHOD 2

Both systems operate at steady state. All kinetic and potential energy effects can be ignored and no significant heat transfer with the surroundings occurs. For each of the two systems, calculate the rate of entropy production in kJ/K per kg of air passing through the system.

SOLUTION:

First analyze METHOD 1.

Apply the 2nd Law to the control volume shown below.



$$\frac{d}{dt}\int_{CV} s\rho dV + \int_{CS} s(\rho \boldsymbol{u}_{rel} \cdot d\boldsymbol{A}) = \int_{CS} \frac{\delta \dot{Q}_{into\ CV}}{T} + \dot{\sigma}, \tag{1}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} s\rho dV = 0 \quad \text{(steady state)}$$

$$\int_{CS} s(\rho \boldsymbol{u}_{rel} \cdot d\boldsymbol{A}) = \dot{m}_{air}(s_{out} - s_{in})$$

$$\int_{CS} \frac{\delta \dot{Q}_{into\ CV}}{T} = 0 \quad \text{(since adiabatic)}$$

Substitute and simplify,

$$s_{out} - s_{in} = \frac{\dot{\sigma}}{\dot{m}_{air}} \tag{2}$$

where σ is the entropy produced per unit mass flow rate.

From thermodynamics tables for air at the given temperatures (note that the air, treated as an ideal gas, is at the same pressure at the inlet and outlet so that: $s(T_B, p_B) - s(T_A, p_A) = s^0(T_B) - s^0(T_A)$):

$s_{out,air}$ = 1.7825 kJ/(kg·K) (T_{out} = 52 °C = 325 K)
$s_{in,air}$ = 1.6680 kJ/(kg·K) (T_{out} = 17 °C = 290 K)

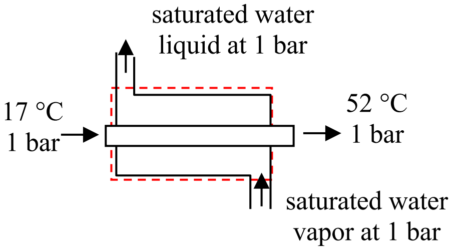
The entropy generation is,

$$\boxed{\frac{\dot{\sigma}}{\dot{m}_{air}} = 0.1145\ \text{kJ/(kg.K)}} \quad \text{for Method 1} \tag{3}$$

Since the entropy production is positive, the process satisfies the Second Law.

Now consider METHOD 2.

Apply the 2nd Law to the control volume shown below.



$$\frac{d}{dt}\int_{CV} s\rho dV + \int_{CS} s(\rho \boldsymbol{u}_{rel}\cdot d\boldsymbol{A}) = \int_{CS}\frac{\delta \dot{Q}_{into\,CV}}{T} + \dot{\sigma} \tag{4}$$

where,

$$\frac{d}{dt}\int_{CV} s\rho dV = 0 \quad \text{(steady state)}$$

$$\int_{CS} s\left(\rho\mathbf{u}_{rel}\cdot d\mathbf{A}\right) = \left[\dot{m}\left(s_{out}-s_{in}\right)\right]_{H_2O} + \left[\dot{m}\left(s_{out}-s_{in}\right)\right]_{air}$$

$$\int_{CS}\frac{\delta\dot{Q}_{into\,CV}}{T} = 0 \quad \text{(no heat transfer into or out of the control volume)}$$

Substitute and simplify,

$$\dot{m}_{H2O}(s_{out}-s_{in})_{H2O} + \dot{m}_{air}(s_{out}-s_{in})_{air} = \dot{\sigma},$$

$$\frac{\dot{m}_{H2O}}{\dot{m}_{air}}(s_{out}-s_{in})_{H2O} + \dot{m}_{air}(s_{out}-s_{in})_{air} = \frac{\dot{\sigma}}{\dot{m}_{air}}, \tag{5}$$

The specific entropies of the outgoing and incoming air were calculated previously. The specific entropies of the saturated liquid and vapor water (both at 1 bar) are found from thermodynamics tables:

$s_{out,H20}$ = 1.3026 kJ/(kg·K) (saturated liquid water at 1 bar)

$s_{in,H20}$ = 7.3594 kJ/(kg·K) (saturated water vapor at 1 bar)

Now apply the First Law to the same control volume to determine the mass flow rate ratio,

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}\rho V^2+gz\right)\left(\rho\mathbf{u}_{rel}\cdot d\mathbf{A}\right) = \dot{Q}_{\substack{into\\CV}} + \dot{W}_{\substack{on\\CV}} \tag{6}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady state)}$$

$$\int_{CS}\left(h+\tfrac{1}{2}\rho V^2+gz\right)\left(\rho\mathbf{u}_{rel}\cdot d\mathbf{A}\right) = \left[\dot{m}\left(h_{out}-h_{in}\right)\right]_{H_2O} + \left[\dot{m}\left(h_{out}-h_{in}\right)\right]_{air}$$

$$\dot{Q}_{\substack{into\\CV}} = \dot{W}_{\substack{on\\CV}} = 0 \quad \text{(adiabatic and no work)}$$

Substitute and simplify,

$$\dot{m}_{H_2O}\left(h_{out}-h_{in}\right)_{H_2O} + \dot{m}_{air}\left(h_{out}-h_{in}\right)_{air} = 0$$

$$\frac{\dot{m}_{H_2O}}{\dot{m}_{air}} = \frac{\left(h_{out}-h_{in}\right)_{air}}{\left(h_{in}-h_{out}\right)_{H_2O}} \tag{7}$$

Using the given data,

$h_{\text{out,air}}$ = 325.31 kJ/kg (T_{out} = 52 °C = 325 K)
$h_{\text{in,air}}$ = 290.16 kJ/kg (T_{out} = 17 °C = 290 K)
$h_{\text{out,H20}}$ = 417.46 kJ/(kg·K) (saturated liquid water at 1 bar)
$h_{\text{in,H20}}$ = 2675.5 kJ/(kg·K) (saturated water vapor at 1 bar)

$$\Rightarrow \frac{\dot{m}_{\text{H}_2\text{O}}}{\dot{m}_{\text{air}}} = 0.0156 \tag{8}$$

Substituting the previous result into Eq. (5) gives,

$$\frac{\dot{\sigma}}{\dot{m}_{air}} = 0.020 \text{ kJ/(kg.K)} \tag{9}$$

Since the entropy production is positive, the data are consistent with the Second Law. Furthermore, the total entropy production per unit mass flow rate of air is 0.020 kJ/(kg·K).

The given results indicate that METHOD 2 is a better approach, thermodynamically speaking, i.e., there is less irreversibility per unit mass flow rate of air, than METHOD 1.