



## **Entropy change for an ideal gas**

(Visit <https://www.nasa.gov/centers/armstrong/multimedia/imagegallery/Schlieren/index.html> for more shock wave photos)

**From the  $T$ - $ds$  relations**

$$Tds = du + pdv \quad \text{and} \quad Tds = dh - vdp$$

**For an ideal gas**

$$pv = RT \Rightarrow vdp + pdv = RdT \quad \text{and} \quad du = c_v(T)dT \quad \text{and} \quad dh = c_p(T)dT$$

Temp. [K]	h [kJ/kg]	u [kJ/kg]	s° [kJ/kg/K]	pr	vr
200	200.0	142.5	1.309	0.3363	1707.0
210	210.0	149.7	1.352	0.3987	1512.0
220	220.0	156.8	1.395	0.4690	1346.0
230	230.0	164.0	1.437	0.5477	1205.0
240	240.0	171.1	1.479	0.6355	1084.0
250	250.0	178.3	1.520	0.7329	979.0
260	260.0	185.4	1.559	0.8405	887.8
270	270.0	192.6	1.597	0.9590	808.0
280	280.1	199.8	1.633	1.0889	738.0

### For the Ideal Gas model

- $p v = R T$  where  $R = \bar{R}_u / M$  and  $\bar{R}_u = 8.314 \text{ kJ}/(\text{kmol}\cdot\text{K})$  and  $M$  is the molecular weight. For example,  $M_{\text{air}} = 28.98 \text{ kg}/\text{kmol} \Rightarrow R_{\text{air}} = 0.287 \text{ kJ}/(\text{kg}\cdot\text{K})$
- $c_p = c_v + R$  and  $k \equiv \frac{c_p}{c_v}$
- $c_v = c_v(T)$ ,  $c_p = c_p(T)$ ,  $u = u(T)$ ,  $h = h(T)$ ,  $s = s(T, p)$
- $u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT$  (find  $u(T)$  in Ideal Gas Tables (IGTs))
- $h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT$  (find  $h(T)$  in IGTs)
- $s(T_2, p_2) - s(T_1, p_1) = s^0(T_2) - s^0(T_1) - R \ln \left( \frac{p_2}{p_1} \right)$  (find  $s^0(T)$  in IGTs)
  - If the process is isentropic, i.e.,  $s_2 = s_1$ , then,
    - $s^0(T_2) - s^0(T_1) = R \ln \left( \frac{p_2}{p_1} \right)$  (find  $s^0(T)$  in IGTs)
    - $\frac{p_2}{p_1} = \frac{p_r(T_2)}{p_r(T_1)}$  (find  $p_r(T)$  in IGTs)
    - $\frac{v_2}{v_1} = \frac{v_r(T_2)}{v_r(T_1)}$  (find  $v_r(T)$  in IGTs)
- If the temperature range isn't too large, then it's reasonable to assume  $c_v, c_p \approx \text{constants}$  (i.e., a perfect gas assumption) and,
  - $u(T_2) - u(T_1) \approx c_v(T_2 - T_1)$ ,
  - $h(T_2) - h(T_1) \approx c_p(T_2 - T_1)$
  - $s(T_2, v_2) - s(T_1, v_1) \approx c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right)$
  - $s(T_2, p_2) - s(T_1, p_1) \approx c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)$
  - If the process is isentropic, i.e.,  $s_2 = s_1$ , then,
    - $\frac{v_2}{v_1} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{1-k}}$      $\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\frac{k}{k-1}}$      $\frac{p_2}{p_1} = \left( \frac{v_2}{v_1} \right)^{-k}$