### 3.4. Thermodynamics Cycles

A cycle is a sequence of processes that begins and ends at the same state. At the conclusion of a cycle, all properties have the same values they had at the beginning of the cycle. Thus, there is no change in the system's state at the end of a cycle. Mathematically,

$$\underbrace{\Delta E_{\text{sys,cycle}}}_{=0} = Q_{\substack{\text{into sys,}\\\text{cycle}}} + W_{\substack{\text{on sys,}\\\text{cycle}}} = Q_{\substack{\text{into sys,}\\\text{cycle}}} - W_{\substack{\text{by sys,}\\\text{cycle}}}, \tag{3.35}$$

or,

$$Q_{\substack{\text{into sys,}\\\text{cycle}}} = -W_{\substack{\text{on sys,}\\\text{cycle}}} \quad \text{or} \quad \boxed{Q_{\substack{\text{into sys,}\\\text{cycle}}} = W_{\substack{\text{by sys,}\\\text{cycle}}}}, \tag{3.36}$$

where the total energy change over the cycle is zero since over a cycle we start and end at the same state. Note that $Q_{\text{into sys,cycle}}$ is the net amount of heat added to the system over the cycle and $W_{\text{by sys, cycle}}$ is the net amount of work done by the system over the cycle.

Cycles are common in many engineering applications. For example, power generation, heat pumps, and refrigeration all involve thermodynamic cycles. Let's consider two general classes of cycles: a power cycle and a refrigeration (or heat pump) cycle.

#### 3.4.1. Power Cycle

In a power cycle, illustrated in Figure 3.10, heat moves from a hot reservoir (part of the surroundings) into the system, which then makes use of the heat to do work on the surroundings, and then ejects the remaining heat to a cold reservoir (another part of the surroundings).

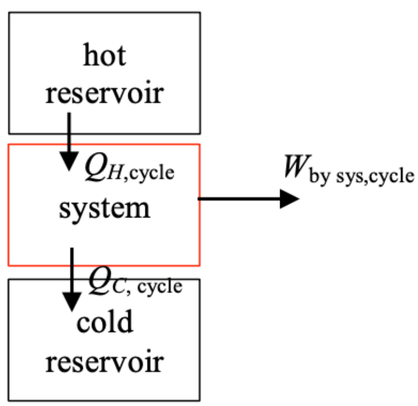



FIGURE 3.10. Illustration of a power cycle.

Utilizing Eq. (3.36) gives,

$$W_{\substack{\text{by sys,}\\\text{cycle}}} = Q_{H,\text{cycle}} - Q_{C,\text{cycle}}, \tag{3.37}$$

where $Q_H$ is the heat transfer interaction with the hot reservoir and $Q_C$ is the heat transfer interaction with the cold reservoir.

*Notes:*

(1) The heat added to the system ($Q_H$) must be greater than the heat removed from the system ($Q_C$) in order for the system to do work on the surroundings.
(2) The heat into the system is generally produced by the combustion of fuel, solar radiation, or a nuclear reaction. The heat out of the system is generally discharged into the surrounding air or a body of water.

(3) The efficiency of the power cycle, $\eta$, is defined as the ratio of the amount of work produced in the cycle to the amount heat added to the system,

$$\eta \coloneqq \frac{W_{\text{by,cycle}}}{Q_{H,\text{cycle}}} = \frac{Q_{H,\text{cycle}} - Q_{C,\text{cycle}}}{Q_{H,\text{cycle}}} = 1 - \frac{Q_{H,\text{cycle}}}{Q_{C,\text{cycle}}}, \tag{3.38}$$

where Eq. (3.37) has been used. The efficiency can never be more than one since the heat out of the system will never be more than the heat into the system over a cycle. We'll discuss the limits on power cycle efficiency in greater detail when discussing the Second Law of Thermodynamics.

(4) The system in the diagram could by any collection of components, e.g., valves, pumps, compressors, turbines, heat exchangers, piston/cylinders, that acts in a cycle. To calculate the efficiency of the cycle, the details of the system don't need to be known - only the heat transfer into the system, the heat transfer out of the system, and the (net) work done by the system need to be known.

### 3.4.2. Refrigeration and Heat Pump Cycles

In refrigeration and heat pump cycles, illustrated in Figure 3.11, heat moves from a cold reservoir (part of the surroundings) into the system, work is done on the system to then eject heat from the system to a hot reservoir (another part of the surroundings). Utilizing Eq. (3.36),

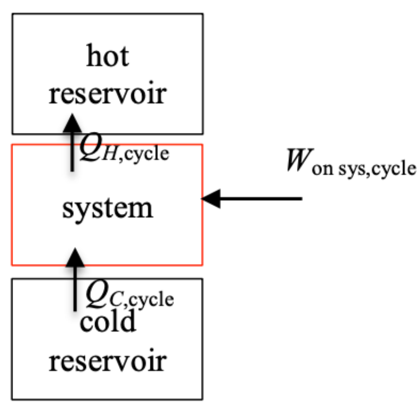



Figure 3.11. Illustration of refrigeration and heat pump cycles.

$$W_{\substack{\text{on sys,}\\ \text{cycle}}} = Q_{H,\text{cycle}} - Q_{C,\text{cycle}}. \tag{3.39}$$

*Notes:*

(1) In refrigeration and heat pump cycles, work is done on the system to make the heat out of the system larger than the heat into the system. Hence, $Q_{H,\text{cycle}} > Q_{C,\text{cycle}}$.

(2) The objective of a refrigeration cycle is to remove heat from the cold reservoir, e.g., a house or refrigerator, to a hot reservoir, e.g., the surrounding environment. A heat pump does the same thing (moves heat from a cold reservoir to a hot reservoir), but the objective is to add heat to the hot reservoir, e.g., remove heat from the environment to raise the temperature in a house. In many of these applications, the power supplied to the system is electrical power.

(3) The system in a typical refrigerator or heat pump is the fluid used within the device. It is what moves the heat (energy, actually) between the cold and hot reservoirs.

(4) Since the goal of a refrigerator is to efficiently remove heat from the cold reservoir, we can define the coefficient of performance ($COP$) of the refrigeration cycle, to be the ratio of the amount heat added to the system from the cold reservoir to the work done on the system over the cycle,

$$COP_{\text{ref}} \coloneqq \frac{Q_{C,\text{cycle}}}{W_{\text{on sys,cycle}}} = \frac{Q_{C,\text{cycle}}}{Q_{H,\text{cycle}} - Q_{C,\text{cycle}}} = \frac{1}{Q_{H,\text{cycle}}/Q_{C,\text{cycle}} - 1}, \tag{3.40}$$

where Eq. (3.39) has been used. Note that the heat out of the system is greater than the heat into the system (since work is done on the system). The refrigeration coefficient of performance can vary from zero to very large values. The larger the $COP_{\text{ref}}$, the larger the transfer of heat from the cold reservoir for a given amount of work over the cycle. The $COP_{\text{ref}}$ for a high efficiency consumer refrigerator is between 1.6 – 3.1. Limits on the value for $COP_{\text{ref}}$ will be discussed further after examining the Second Law of Thermodynamics.

(5) The goal of a heat pump is to efficiently move heat into a hot reservoir; hence, we can define the coefficient of performance ($COP$) of the heat pump cycle, as the ratio of the amount heat added to the hot reservoir to the work done on the system over the cycle,

$$COP_{\text{HP}} \coloneqq \frac{Q_{H,\text{cycle}}}{W_{\text{on sys,cycle}}} = \frac{Q_{H,\text{cycle}}}{Q_{H,\text{cycle}} - Q_{C,\text{cycle}}} = \frac{1}{1 - Q_{C,\text{cycle}}/Q_{H,\text{cycle}}}, \tag{3.41}$$

where Eq. (3.39) has been used. The heat pump coefficient of performance can never be less than one since the heat out of the system always be larger than the heat into the system (work is done on the system). The larger the $COP_{\text{HP}}$, the larger the transfer of heat for a given amount of work over the cycle. A typical $COP_{\text{HP}}$ for a commercial heat pump is between 3 – 4. Limits on the value for $COP_{\text{HP}}$ will be discussed further after examining the Second Law of Thermodynamics.

(6) In the U.S., refrigeration and heat pump $COP$s are often expressed as Energy Efficiency Ratios, $EER$s, which are simply $COP$s, but with an unfortunate mix of English and SI units,

$$EER \coloneqq \frac{\dot{Q} \quad \text{in Btu/h}}{\dot{W}_{\text{on sys,cycle}} \quad \text{in W}}. \tag{3.42}$$

Using appropriate unit conversions,

$$COP = \left(0.292\,\frac{\text{W}}{\text{Btu/h}}\right) EEP. \tag{3.43}$$

(7) Note that Eqs. (3.36) - (3.41) may all be written in rates of change too,

$$\dot{Q}_{\substack{\text{into sys,}\\ \text{cycle}}} = -\dot{W}_{\substack{\text{on sys,}\\ \text{cycle}}} \quad \text{or} \quad \dot{Q}_{\substack{\text{into sys,}\\ \text{cycle}}} = \dot{W}_{\substack{\text{by sys,}\\ \text{cycle}}} \tag{3.44}$$

$$\dot{W}_{\substack{\text{by sys,}\\ \text{cycle}}} = \dot{Q}_{H,\text{cycle}} - \dot{Q}_{C,\text{cycle}}, \tag{3.45}$$

$$\eta \coloneqq \frac{\dot{W}_{\text{by,cycle}}}{\dot{Q}_{H,\text{cycle}}}, \tag{3.46}$$

$$\dot{W}_{\substack{\text{on sys,}\\ \text{cycle}}} = \dot{Q}_{H,\text{cycle}} - \dot{Q}_{C,\text{cycle}}, \tag{3.47}$$

$$COP_{\text{ref}} \coloneqq \frac{\dot{Q}_{C,\text{cycle}}}{\dot{W}_{\text{on sys,cycle}}}, \tag{3.48}$$

$$COP_{\text{HP}} \coloneqq \frac{\dot{Q}_{H,\text{cycle}}}{\dot{W}_{\text{on sys,cycle}}}. \tag{3.49}$$

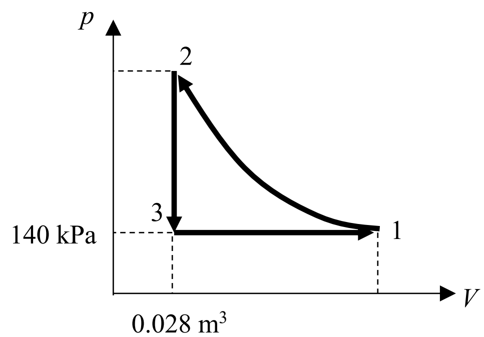
A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series:

Process 1 – 2: Compression with constant internal energy ($pV$ = constant)
Process 2 – 3: Constant volume cooling to a pressure of 140 kPa (abs) and a volume of 0.028 m$^3$
Process 3 – 1: Constant pressure expansion with a total work of 10.5 kJ acting on the piston

For the cycle, the net amount of work done by the gas on the piston is -8.3 kJ. There are no changes in kinetic or potential energy.

a. Sketch the processes on a $p$-$V$ diagram.
b. Determine the volume at state 1, in m$^3$.
c. Determine the work and heat transfer for process 1 – 2, each in kJ.
d. Is this a power cycle or a refrigeration/heat pump cycle? Explain.

SOLUTION:




The volume at state 1 may be found by knowing that the work in going from state 3 to state 1 is 10.5 kJ,

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ 3\to 1}} = \int_3^1 p\,dV = p\int_{V=V_3}^{V=V_1} dV = p\left(V_1 - V_3\right), \quad \text{(since the pressure is constant from 3 to 1)} \tag{1}$$

$$V_1 = V_3 + \frac{W_{\substack{\text{by gas}\\ \text{on piston,}\\ 3\to 1}}}{p}. \tag{2}$$

Using the given parameters,

$V_3$ = 0.028 m$^3$
$W_{\substack{\text{by gas}\\ \text{on piston,}\\ 3\to 1}}$ = 10.5 kJ
$p$ = 140 kPa (abs)

$$\Rightarrow \boxed{V_1 = 0.103 \text{ m}^3} \tag{3}$$

The work in going from state 1 to state 2 can be found by knowing that the total work done by the gas on the piston over the whole cycle is -8.3 kJ, because the volume remains constant in going from state 2 to state 3, the corresponding work is zero, and the work on the piston in going from state 3 to state 1 is 10.5 kJ,

$$\underbrace{W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{cycle}}}}_{=-8.5\text{ kJ}} = W_{\substack{\text{by gas}\\ \text{on piston,}\\ 1\to 2}} + \underbrace{W_{\substack{\text{by gas}\\ \text{on piston,}\\ 2\to 3}}}_{=0} + \underbrace{W_{\substack{\text{by gas}\\ \text{on piston,}\\ 3\to 1}}}_{=10.5\text{ kJ}}, \tag{4}$$

$$\boxed{W_{\substack{\text{by gas}\\ \text{on piston,}\\ 1\to 2}} = -18.8 \text{ kJ}}. \tag{5}$$

The heat transferred in the process from state 1 to state 2 can be found using the 1st Law of Thermodynamics and noting that the energy remains unchanged in going from 1 to 2,

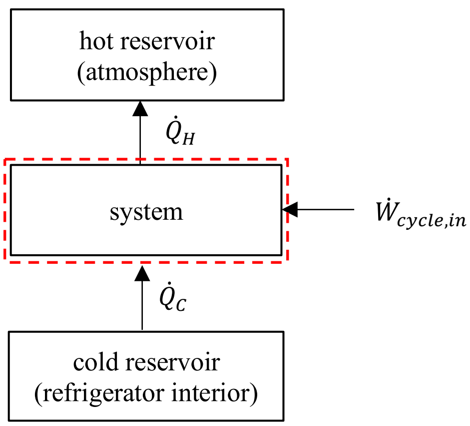
$$\underbrace{\Delta E_{\substack{\text{gas,}\\1\to2}}}_{=0} = Q_{\substack{\text{into gas,}\\1\to2}} - \underbrace{W_{\substack{\text{by gas,}\\1\to2}}}_{=-18.8\text{ kJ}} \quad \Rightarrow \quad \boxed{Q_{\substack{\text{into gas,}\\1\to2}} = -18.8\text{ kJ}} \tag{6}$$

Since $W_{\text{by gas,cycle}}$ = -8.3 kJ < 0, this is a refrigeration (or heat pump) cycle.

A refrigerator steadily receives a power input of 0.15 kW while rejecting energy by heat transfer to the surroundings at a rate of 0.6 kW.

a. Determine the rate at which energy is removed by heat transfer from the refrigerated space.
b. Determine the refrigerator's coefficient of performance.

SOLUTION:




Apply the 1st Law to the system to determine the rate at which heat is transferred from the refrigerator interior into the system,

$$\dot{W}_{cycle,in} = \dot{Q}_H - \dot{Q}_C, \tag{1}$$

$$\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle,in}. \tag{2}$$

Using the given data,

$\dot{Q}_H = 0.6$ kW,

$\dot{W}_{cycle,in} = 0.15$ kW,

⇨ $\underline{\dot{Q}_C = 0.45 \text{ kW}}$.

The coefficient of performance for a refrigeration cycle is,

$$COP_{ref} = \frac{\dot{Q}_C}{\dot{W}_{cycle,in}}. \tag{3}$$

Using the given data,

$\boxed{COP_{ref} = 3.0}$.