3.2.3. Heat

Heat is another form of boundary energy interaction occurring between a system and its surroundings. The difference between heat and work is that heat transfer occurs due to differences in temperature and work occurs through mechanical or electrical means. Heat moves from regions of high temperature to regions of low temperature. Like work, heat is not a property of a system but rather is associated with a process. The amount of heat transferred during a process depends on the path taken during the process. To signify its path dependence, the small amount of heat transferred in a process is signified using the inexact differential, δQ.

Notes:

(1) Specific heat transfer, q, is the heat transfer per unit mass, e.g., $q = Q/m$. Example units would be kJ/kg.

Heat can be transferred between the system and surroundings via three methods: conduction, convection, and radiation.

3.2.3.1. Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can occur in any substance: solids, liquids, and gases. In gases and liquids, conduction occurs due to collisions between molecules during their random motion. In solids, conduction occurs as a result of molecular vibrations and electron transfer. As the more energetic particles collide or contact the less energetic particles, there is a transfer of energy causing an increase in the energy of the less energetic particles and a decrease in energy of the more energetic particles.

The temperature of a region containing many molecules is a measure of the energy due to random translational, rotational, and vibrational motions of the molecules. The larger the temperature, the more random energy the molecules have. Thus, conduction or the transfer of energy due to molecular interactions, will occur from regions of high temperature to regions of low temperature.

The rate of heat transfer, $\dot{\mathbf{Q}}$, (this is a vector quantity since the heat energy travels in a particular direction) due to conduction through an area, A, of a substance is given by Fourier's Law of Heat Conduction,

$$\dot{\mathbf{Q}} = -kA\boldsymbol{\nabla}T, \tag{3.27}$$

where k is a material property of the substance known as the thermal conductivity, and $\boldsymbol{\nabla}T$ is the temperature gradient in the substance. Note that the negative sign in the equation is required so that heat moves from regions of higher temperature to regions of lower temperature.

The thermal conductivity is a measure of how well a material can conduct heat energy. Materials that conduct heat energy well have large k, e.g., $k_{\text{diamond}} = 2300\,\text{W}\,\text{m}^{-1}\,\text{K}$, and those that conduct heat energy poorly have small k, e.g., $k_{\text{air}} = 0.026\,\text{W}\,\text{m}^{-1}\,\text{K}$.

3.2.3.2. Convection

Convection is the mode of energy transfer between and a solid surface and an adjacent fluid that is in motion; it involves the combined effects of conduction and relative fluid motion (also known as advection).

Convection can be further classified as forced convection or free (or natural) convection. In forced convection, the fluid motion is produced via external means, e.g., by a fan or pump. In free convection, the fluid motion is induced by buoyant forces arising from density difference in the fluid caused by temperature variations.

The rate of heat transfer, $\dot{Q}$, leaving a surface with area, A_s, and entering the fluid due to convection is given by Newton's Law of Cooling,

$$\dot{Q} = hA_s\left(T_s - T_f\right), \tag{3.28}$$

where h is the heat transfer coefficient for the system and T_s is the temperature of the surface that is in contact with the fluid with temperature, T_f. The heat transfer coefficient depends on the surface and fluid properties as well as the flow characteristics. It is generally an experimentally determined property for all but the simplest flow situations. Typical ranges for the free convection heat transfer coefficient are $2-25\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $50-1000\,\mathrm{W\,m^{-1}\,K^{-1}}$ for gases and liquids, respectively. For forced convection, the range is $25-250\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $50-20000\,\mathrm{W\,m^{-1}\,K^{-1}}$ for gases and liquids, respectively.

3.2.3.3. Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves as a result of changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, radiation does not require an intervening medium for transferring heat.

The rate at which heat is emitted from a surface with area, A_s, depends on the *absolute* temperature of the surface, T_s, as indicated by the modified Stefan-Boltzmann Law,

$$\dot{Q}_{\mathrm{emitted}} = \epsilon\sigma A_s T_s^4, \tag{3.29}$$

where $\dot{Q}_{\mathrm{emitted}}$ is the rate at which heat is emitted from the surface, ϵ is the emissivity of the surface ($0 \le \epsilon \le 1$), and σ is the Stefan-Boltzmann constant ($\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-4}}) = 0.1714\times10^{-8}\,\mathrm{Btuh^{-1}ft^{-2}{}^\circ R^{-4}}$)).

A blackbody is an object with an emissivity of one, $\sigma_{\mathrm{blackbody}} = 1$, i.e., a blackbody is a perfect emitter of radiation.

Surfaces can also absorb radiation. The heat flux absorbed by a surface via radiation is given by,

$$\dot{Q}_{\mathrm{absorbed}} = \alpha\dot{Q}_{\mathrm{incident}}, \tag{3.30}$$

where α is the absorptivity of the surface ($0 \le \alpha \le 1$). Note that a blackbody is defined as having $\alpha = 1$ making it is both a perfect emitter and perfect absorber of radiation.

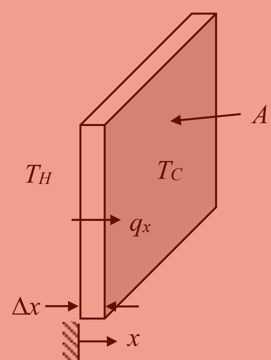
Actual determination of the rate at which radiation is emitted and absorbed by a surface can be complicated since the rate depends on factors such as surface orientation, the effects of the intervening medium, and the surface spectral characteristics.

For the special case in which a small surface interacts with a much larger surface, the intervening fluid has no affect on the radiation transfer, and $\alpha = \sigma$ (termed a grey body), the rate of heat transfer from the surface to the surroundings via radiation is,

$$\dot{Q}_{\mathrm{emitted}} = \epsilon\sigma A_s\left(T_s^4 - T_{\mathrm{surr}}^4\right). \tag{3.31}$$

An insulated frame wall of a house has an average thermal conductivity of 0.0318 Btu/(hr·ft·°R). The thickness of the wall is 6 in. At steady state, the rate of energy transfer by conduction through an area of 160 ft^2 is 400 Btu/hr, and the temperature decreases linearly from the inner surface to the outer surface. If the outside surface temperature of the wall is 30 °F, what is the inner surface temperature in °F?

SOLUTION:



k = 0.0318 Btu/(hr·ft·°R)
Δx = 6 in = 0.5 ft
A = 160 ft^2
$\dot{Q}$ = 400 Btu/hr
T_C = 30 °F = 490 °R

From Fourier's Law, the heat transfer through the wall is:

$$\dot{Q}_x = -kA\frac{dT}{dx} \approx -kA\frac{T_C - T_H}{\Delta x} \quad (1)$$

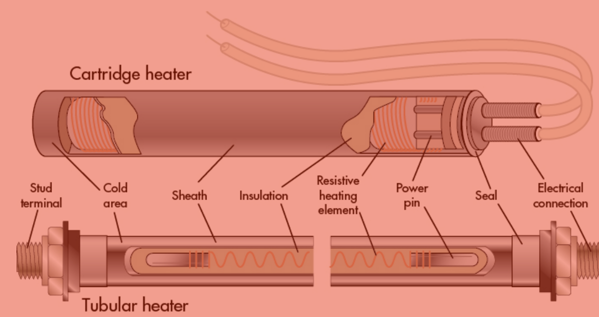
Re-arrange to solve for T_H.

$$\boxed{T_H = T_C + \frac{\dot{Q}}{kA}\Delta x} \quad (2)$$

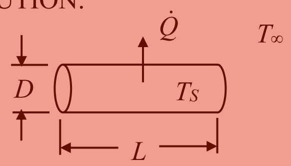
Using the given parameters:

k = 0.0318 Btu/(hr·ft·°R)
Δx = 6 in = 0.5 ft
A = 160 ft^2
$\dot{Q}$ = 400 Btu/hr
T_C = 30 °F = 490 °R
$\Rightarrow$ $\boxed{T_H = 529\ °R = 69\ °F}$

A cartridge electrical heater is shaped as a cylinder of length 200 mm and outer diameter of 20 mm. Under normal operating conditions the heater dissipates 2 kW while submerged in a water flow which is at 20 °C and provides a convection heat transfer coefficient of 5000 W/(m^2·K). Neglecting heat transfer from the ends of the heater, determine the heater's surface temperature. If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air which is also at 20 °C but for which the heat transfer coefficient is 50 W/(m^2·K). What is the corresponding surface temperature? What are the consequences of such an event?



SOLUTION:



Determine the surface temperature using Newton's Law of Cooling.

$$\dot{Q} = hA\left(T_S - T_\infty\right) \quad (1)$$

where $A = \pi DL$. Re-arranging gives:

$$\boxed{T_S = T_\infty + \frac{\dot{Q}}{hA}} \quad (2)$$

Using the given parameters:

T_∞ = 20 °C = 293 K
$\dot{Q}$ = 2000 W
h = 5000 W/(m^2·K)
D = 20*10^{-3} m
L = 200*10^{-3} m
$\Rightarrow$ A = 1.3*10^{-2} m^2
$\boxed{T_S = 325\text{ K} = 52\text{ °C}}$

If instead, h = 50 W/(m^2·K), then:

$\boxed{T_S = 3500\text{ K} = 3200\text{ °C}}$

This temperature is probably large enough to melt the cartridge heater!

An uninsulated steam pipe passes through a room in which the air and walls are at 25 °C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200 °C and 0.8, respectively. If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/(m^2·K), what is the rate of heat loss from the surface per unit length of pipe?

SOLUTION:

The convective heat transfer rate, $\dot{Q}_C$, is given by Newton's Law of Cooling:

$$\dot{Q}_C = hA_S\left(T_S - T_\infty\right) \tag{1}$$

where h is the convection heat transfer coefficient, A_S is the surface area of the pipe, T_S is the surface temperature of the pipe and T_∞ is the ambient temperature.

The radiative heat transfer rate, $\dot{Q}_R$, is given by:

$$\dot{Q}_R = \varepsilon\sigma A_S\left(T_S^4 - T_\infty^4\right) \tag{2}$$

where ε is the surface emissivity and σ is the Stefan-Boltzmann constant.

The total heat transfer rate from the pipe is:

$$\dot{Q}_T = \dot{Q}_C + \dot{Q}_R \tag{3}$$

Using the given parameters:

h = 15 W/(m^2·K)
D = 70*10^{-3} m $\Rightarrow$ $A_S = \pi DL$ $\Rightarrow$ A_S/L = 0.22 m
T_S = 200 °C = 473 K
T_∞ = 15 °C = 288 K
ε = 0.8
σ = 5.67*10^{-8} W/(m^2·K^4)
$\Rightarrow$ $\dot{Q}_C / L$ = 580 W/m
$\dot{Q}_R / L$ = 420 W/m
$\boxed{\dot{Q}_T / L = 1 \text{ kW/m}}$

3.3. The First Law of Thermodynamics

In words and in mathematical form, the First Law of Thermodynamics is: The increase in total energy of a system is equal to the energy added to the system via heat transfer plus the energy added to the system via work done on the system,

$$dE_{\text{sys}} = \delta Q_{\text{into sys}} + \delta W_{\text{on sys}}, \tag{3.32}$$

where dE_{sys} is a small increase in the total energy of the system, $\delta Q_{\text{into sys}}$ is a small amount of energy transferred into the system via heat transfer, and $\delta W_{\text{on sys}}$ is a small amount of energy added to the system via work done on the system by the surroundings (Figure 3.9). Note that work and heat are just methods of transferring energy, hence, the First Law of Thermodynamics can also be thought of as Conservation of Energy.

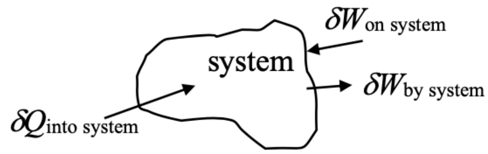


Figure 3.9. A schematic showing a system and the directions of energy transfer.

Notes:

(1) Since energy is a property of a system, an exact differential (the "d" operator in dE) is used to specify the small change in the energy. In other words, the difference in energy between two states depends only upon the endpoint states and is independent of the path between the two states. The small change in heat and work are indicated using an inexact differential (the "δ" operator in δQ and δW) to signify that both heat and work are path dependent processes.

(2) Note that different disciplines have different notations for the First Law. In particular, in thermodynamics, work is usually discussed in terms of the work done *by* the system on the surroundings so that the First Law becomes,

$$dE_{\text{sys}} = \delta Q_{\text{into sys}} - \delta W_{\text{by sys}}. \tag{3.33}$$

In order to avoid confusion regarding the proper sign for work, these notes will try to clearly specify whether work is being done on or by the system. Understanding that if one does work on a system, the system's energy will increase is generally sufficient to avoid most sign convention problems.

(3) We can also write the First Law in terms of time rates of changes by taking the limit of the changes in the properties over a short amount of time as the time duration approaches zero,

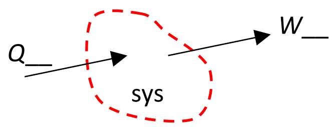
$$\frac{dE_{\text{sys}}}{dt} = \delta \dot{Q}_{\text{into sys}} + \delta \dot{W}_{\text{on sys}} = \delta \dot{Q}_{\text{into sys}} - \delta \dot{W}_{\text{by sys}}. \tag{3.34}$$

Notes:

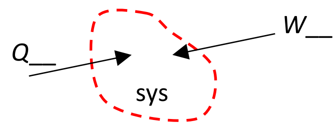
(1) Recall that <u>specific work, w</u>, is the work per unit mass, i.e., $w = W/m$. It can also be written as the power per unit mass flow rate, i.e., $w = \dot{W}/\dot{m}$. Similarly, <u>specific heat transfer, q</u>, can be written as Q/m or $\dot{Q}/\dot{m}$. Example units for specific work and specific heat transfer are kJ/kg.

Now let's consider a few simple examples.

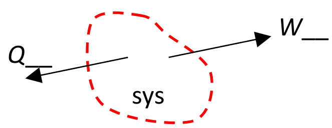
Fill in the subscripts in the following Energy Flow Diagrams (e.g., on, by, in, out) and the subscripts and signs for the corresponding First Law expression.



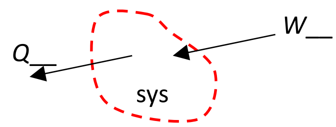
$\Delta E_{sys} = \_\_Q\_\_\ \_\_\ W\_\_$



$\Delta E_{sys} = \_\_Q\_\_\ \_\_\ W\_\_$

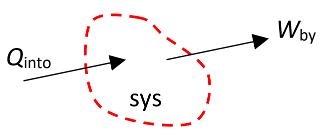


$\Delta E_{sys} = \_\_Q\_\_\ \_\_\ W\_\_$

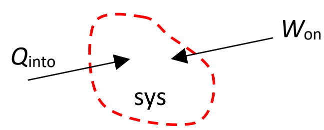


$\Delta E_{sys} = \_\_Q\_\_\ \_\_\ W\_\_$

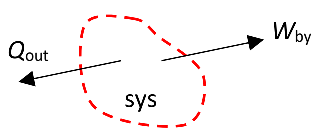
SOLUTION:



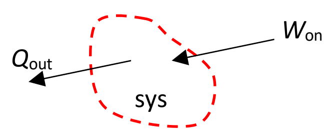
$\Delta E_{sys} = +Q_{into} - W_{by}$



$\Delta E_{sys} = +Q_{into} + W_{on}$



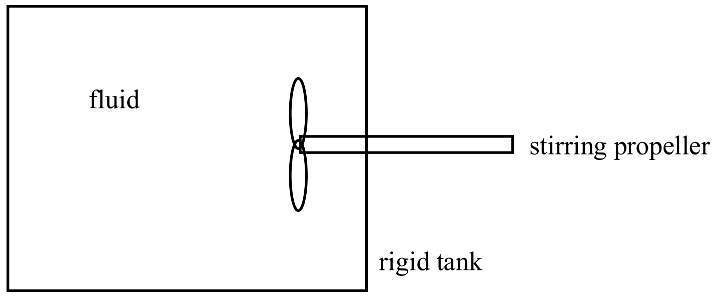
$\Delta E_{sys} = -Q_{out} - W_{by}$



$\Delta E_{sys} = -Q_{out} + W_{on}$

To avoid ambiguity and confusion, always add a subscript to your Q and W terms in your Energy Flow Diagrams and in your First Law expression. In addition, make sure the arrows, subscripts, and equation are all consistent.

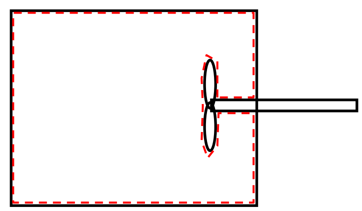
A rigid tank contains a hot fluid that is cooled while being stirred. Initially the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat and the stirring propeller does 100 kJ of work on the fluid. What is the final internal energy of the fluid?



SOLUTION:

Apply the 1st Law to the system of fluid contained within the tank.

$$\Delta E = E_f - E_i = Q_{\text{into system}} + W_{\text{on system}}$$



where

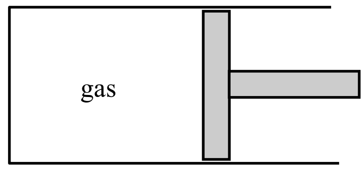
$E_i = U_i = 800$ kJ

$Q_{\text{into system}} = -500$ kJ

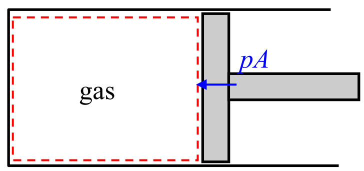
$W_{\text{on system}} = 100$ kJ

$\Rightarrow \boxed{E_f = U_f = 400 \text{ kJ}}$

Four kilograms of a certain gas is contained within a piston-cylinder assembly. The gas undergoes a polytropic process where: $pV^{1.5}$=constant. The initial pressure is 3 bars (abs), the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. The change in the specific internal energy of the gas in the process is Δu = -4.5 kJ/kg. There are no significant changes in the kinetic or potential energies of the gas. What is the net heat transfer for the process?



SOLUTION:
Apply the First Law to the system of gas as shown in the figure below,



$$\Delta E_{\text{sys}} = Q_{\substack{\text{added}\\ \text{to sys}}} + W_{\text{on sys}}, \tag{1}$$

where,

$$W_{on\ sys} = \int_{V=V_1}^{V=V_2} -p dV = \int_{V=V_1}^{V=V_2} -(cV^{-1.5}) dV = 2c(V^{-0.5})_{V_1}^{V_2} = 2 \underbrace{p_1 V_1^{1.5}}_{=c} (V_2^{-0.5} - V_1^{-0.5}), \tag{2}$$

and,

$$\Delta E_{\text{sys}} = m_{\text{sys}} \Delta e_{\text{sys}} = m_{\text{sys}} \Delta u_{\text{sys}}. \quad \text{(The kinetic and potential energy changes are negligible.)} \tag{2}$$

Re-arranging Eq. (1) and substituting Eqs. (2) and (3) gives,

$$\boxed{Q_{\substack{added\\ to\ sys}} = m_{sys} \Delta u_{sys} - 2 p_1 V_1^{1.5} (V_2^{-0.5} - V_1^{-0.5})}. \tag{3}$$

Using the given values:

m_{sys} = 4 kg
Δu_{sys} = -4500 J/kg
p_1 = 3*10^5 Pa
V_1 = 0.1 m^3
V_2 = 0.2 m^3
$\Rightarrow$ $\boxed{Q_{\text{added}} = -0.426 \text{ kJ}}$ (heat is leaving the system)

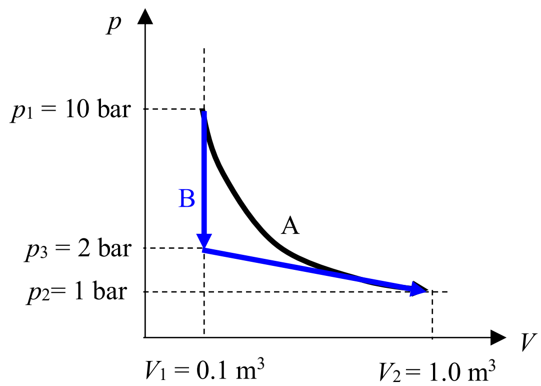
A gas contained within a piston-cylinder assembly undergoes two processes, A and B, between the same end states, 1 and 2, where at state 1 the pressure is 10 bar, the volume is 0.1 m^3, the internal energy is 400 kJ, and at state 2 the pressure is 1 bar, the volume is 1.0 m^3, and the internal energy is 200 kJ.

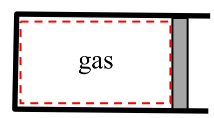
Process A: Process from 1 to 2 during which the pressure-volume relation is pV = constant.
Process B: Constant volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.

Kinetic and potential energy effects can be ignored. For each of the processes A and B,

a. Sketch the process on a p-V diagram,
b. evaluate the work by the gas on the piston, in kJ, and
c. evaluate the heat transfer from the gas in kJ.

SOLUTION:
The processes are sketched on the plot shown below.





The work may be found by integrating the pdV work given the two processes described,

$$W_{\substack{\text{by gas}\\ \text{on piston}}} = \int_1^2 p\,dV\,, \tag{1}$$

where for process A,

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path A}}} = c\int_{V=V_1}^{V=V_2} \frac{dV}{V} = c\ln\left(\frac{V_2}{V_1}\right), \tag{2}$$

noting that $pV = c \Rightarrow p = c/V$. The constant c may be found from the initial (or final) conditions,

$$p_1V_1 = c = p_2V_2\,, \tag{3}$$

$$W_{\substack{\text{by gas}\\ \text{on piston,}\\ \text{path A}}} = p_1V_1\ln\left(\frac{V_2}{V_1}\right). \tag{4}$$

Substituting the given numbers,

p_1 = 10 bar = 10*10^2 kPa
V_1 = 0.1 m^3
V_2 = 1.0 m^3

$$\Rightarrow \boxed{W_{\text{by gas on piston, path A}} = 230\text{ kJ}} \tag{5}$$

The heat transferred from the gas may be found using the 1st Law of Thermodynamics,

$$\underbrace{\Delta E_{\text{gas}}}_{=\Delta U_{\text{gas}}} = Q_{\substack{\text{into}\\\text{gas}}} - W_{\substack{\text{by}\\\text{gas}}} \Rightarrow Q_{\substack{\text{into}\\\text{gas}}} = \Delta U_{\text{gas}} + W_{\substack{\text{by}\\\text{gas}}}, \quad (6)$$

where the total change of energy in the gas is due only to changes in internal energy (U). Using the given parameters,

U_1 = 400 kJ
U_2 = 200 kJ
$W_{\text{by gas}}$ = 230 kJ (from Eq. (5))

$$\Rightarrow \Delta U = -200\text{ kJ} \Rightarrow \boxed{Q_{\text{into gas}} = 30\text{ kJ}} \quad (7)$$

Thus, 30 kJ of heat is transferred into the gas (-30 kJ of heat is transferred from the gas).

For process B, there is no work done in the constant volume part of the process since the volume doesn't change. The work in the linear pressure-volume part of the process is,

$$W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path B}}} = \int_{V=V_1}^{V=V_2} p\,dV = \int_{V=V_1}^{V=V_2}\left[\left(\frac{p_2-p_3}{V_2-V_1}\right)(V-V_1)+p_3\right]dV = \left[\left(\frac{p_2-p_3}{V_2-V_1}\right)\left(\tfrac{1}{2}V^2-V_1V\right)+p_3V\right]_{V_1}^{V_2}, \quad (8)$$

$$W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path B}}} = \left(\frac{p_2-p_3}{V_2-V_1}\right)\left(\tfrac{1}{2}V_2^2-\tfrac{1}{2}V_1^2-V_1V_2+V_1^2\right)+p_3(V_2-V_1), \quad (9)$$

$$W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path B}}} = \tfrac{1}{2}\left(\frac{p_2-p_3}{V_2-V_1}\right)\left(V_2^2-2V_1V_2+V_1^2\right)+p_3(V_2-V_1), \quad (10)$$

where the pressure varies linearly with the volume,

$$p = \left(\frac{p_2-p_3}{V_2-V_1}\right)(V-V_1)+p_3, \text{ (equation of a line)} \quad (11)$$

Substituting the given data,

p_2 = 1 bar = 1*10^2 kPa
p_3 = 2 bar = 2*10^2 kPa
V_1 = 0.1 m^3
V_2 = 1.0 m^3

$$\Rightarrow \boxed{W_{\substack{\text{by gas}\\\text{on piston,}\\\text{path B}}} = 135\text{ kJ}} \quad (12)$$

The heat transferred into the gas may be found using Eq. (6) with the following parameters,

U_1 = 400 kJ (Note that the internal energies are independent of the path. They're a property!)
U_2 = 200 kJ
$W_{\text{by gas}}$ = 135 kJ (from Eq. (12))

$$\Rightarrow \boxed{Q_{\text{into gas}} = -65\text{ kJ}} \quad (13)$$

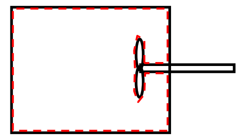
Thus, 65 kJ of heat is transferred from the gas to the surroundings.

A gas is contained in a closed rigid tank fitted with a paddle wheel. The paddle wheel stirs the gas for 20 min, with the power varying with time *t* according to (10 W/min)*t*. Heat transfer from the gas to the surroundings takes place at a constant rate of 50 W. Determine:

a. the rate of change of energy of the gas at time 10 min, in watts, and
b. the net change in energy of the gas after 20 min, in kJ.

SOLUTION:

Apply the 1st Law to the system of gas within the tank,



$$\frac{dE_{\text{sys}}}{dt} = \dot{Q}_{\substack{\text{added}\\\text{to sys}}} + \dot{W}_{\text{on sys}}, \tag{1}$$

where $\dot{Q}_{\text{into sys}} = -50\text{ W}$ and $\dot{W}_{\text{on sys}} = \left(10\,{}^{\text{W}}\!/\!_{\text{min}}\right)t$. Thus, at *t* = 10 min,

$$\frac{dE_{\text{sys}}}{dt} = -50\text{ W} + \left(10\,{}^{\text{W}}\!/\!_{\text{min}}\right)t, \tag{2}$$

$$\boxed{\therefore \frac{dE_{\text{sys}}}{dt} = 50\text{ W}}.$$

The net change in energy of the gas is found by integrating Eqn. (2) in time,

$$\Delta E_{\text{sys}} = \int_{t=0}^{t=20\text{ min}} \frac{dE_{\text{sys}}}{dt}dt = \int_{t=0}^{t=20\text{ min}} \left[50\text{ W} + \left(10\,{}^{\text{W}}\!/\!_{\text{min}}\right)t\right]dt = \left[\left(50\text{ W}\right)t + \tfrac{1}{2}\left(10\,{}^{\text{W}}\!/\!_{\text{min}}\right)t^2\right]_{t=0}^{t=20\text{ min}} \tag{3}$$

$$\boxed{\therefore \Delta E_{\text{sys}} = 60\text{ kJ}}$$