

## 1.7. Fluid Properties

Some good fluid property references include:

- Avallone, E.A. and Baumeister III, T., *Marks' Standard Handbook for Mechanical Engineers*, McGraw-Hill.
- Kestin, J., and Wakeham, W.A., *Transport Properties of Fluids*, CINDAS Data Series on Material Properties, C.Y. Ho, ed., Hemisphere Publishing.

### 1.7.1. Density, $\rho$

- The density of a substance is a measure of how much mass there is of the substance per unit volume.
- The dimensions of density are  $M/L^3$ . Typical units are  $\text{kg/m}^3$ ,  $\text{slug/ft}^3$ , and  $\text{lb}_m/\text{ft}^3$ .
- The density of water at  $4^\circ\text{C}$  is  $\rho_{\text{H}_2\text{O},4^\circ\text{C}} = 1000 \text{ kg/m}^3 = 1.94 \text{ slug/ft}^3 = 62.4 \text{ lb}_m/\text{ft}^3$ .
- The density of air at Standard Temperature ( $= 15^\circ\text{C}$ ) and Pressure ( $= 101.3 \text{ kPa (abs)}$ ) (STP) is  $\rho_{\text{air, STP}} = 1.23 \text{ kg/m}^3 = 2.38 \times 10^{-3} \text{ slug/ft}^3 = 7.68 \times 10^{-2} \text{ lb}_m/\text{ft}^3$ .
- Density does not vary greatly with temperature for liquids, in general.
- Density does change considerably with temperature and pressure for gases.
- A substance where the density remains constant for all conditions is considered incompressible.
  - A good engineering rule of thumb is that if there are no significant temperature changes and for fluid velocities less than approximately  $1/3$  the speed of sound in the fluid, the fluid can be approximated as incompressible (the proof of this is examined in Chapter 13).
  - In air, the speed of sound at standard conditions ( $p = 1 \text{ atm (abs)}$ ,  $T = 59^\circ\text{F} = 15^\circ\text{C}$ ), is approximately  $1100 \text{ ft s}^{-1}$  ( $340 \text{ m s}^{-1}$ ).
  - The speed of sound in water is approximately  $4800 \text{ ft s}^{-1}$  ( $1500 \text{ m s}^{-1}$ ).
  - The speed of sound in steel is approximately  $16\,400 \text{ ft s}^{-1}$  ( $5000 \text{ m s}^{-1}$ ).
  - In most instances, liquids and gases flowing at low speeds can be approximated as incompressible.
- specific gravity,  $SG$ 
  - Specific gravity is dimensionless. It has no units.
  - The specific gravity of a liquid is the ratio of the liquid's density to the density of water at some specified condition, typically at  $4^\circ\text{C}$ ,

$$SG_{\text{liquid}} := \frac{\rho_{\text{liquid}}}{\rho_{\text{H}_2\text{O},4^\circ\text{C}}}. \quad (1.74)$$

For example, the density of mercury (Hg) at  $20^\circ\text{C}$  is  $13.6 \times 10^3 \text{ kg/m}^3$ . Hence,  $SG_{\text{Hg}} = 13.6$ .

- Specific weight,  $\gamma$ 
  - The specific weight of a substance is the weight of the substance per unit volume,
  - The dimensions of specific weight are  $F/L^3$  and common units are  $\text{N/m}^3$  and  $\text{lb}_f/\text{ft}^3$ .

$$\gamma := \frac{W}{V} = \frac{mg}{V} = \frac{\rho V g}{V} = \rho g. \quad (1.75)$$

For example, the specific weight of water at  $4^\circ\text{C}$  is  $9.81 \times 10^3 \text{ N/m}^3 = 62.4 \text{ lb}_f/\text{ft}^3$ .

- Specific volume,  $v$ 
  - The specific volume of a substance is how much volume the substance occupies per unit mass.
  - The dimensions of specific volume are  $L^3/M$ . Common units are  $\text{m}^3/\text{kg}$ ,  $\text{ft}^3/\text{slug}$ , and  $\text{ft}^3/\text{lb}_m$ .
  - The specific volume is simply the inverse of the density,

$$v = \frac{1}{\rho}. \quad (1.76)$$

- In thermodynamics, the specific volume is commonly used in place of density.

*Example:*

What is the specific weight of water if its density is  $62.4 \text{ lb}_m/\text{ft}^3$ ?

*Solution:*

$$\gamma = \rho g = (62.4 \text{ lb}_m/\text{ft}^3) (32.2 \text{ ft/s}^2) \left( \frac{1 \text{ lb}_f}{32.2 \text{ lb}_m \text{ ft/s}^2} \right) = 62.4 \text{ lb}_f/\text{ft}^2 \quad (1.77)$$

Be sure to:

- (1) Be careful when using the incompressible flow assumption. Make sure that the assumption is reasonable for your flow situation. For liquids, the assumption is usually reasonable. For gases, you need to check the flow velocities and temperatures.

### 1.7.2. Pressure, $p$

- The dimensions of pressure are  $F/L^2$ . Common units are Pa = N/m<sup>2</sup>, psf = lb<sub>f</sub>/ft<sup>2</sup>, psi = lb<sub>f</sub>/in<sup>2</sup>, torr, atm, bar, and mmHg and inHg.
- Pressure is a scalar quantity. It has a magnitude, but no direction.
- The differentially small pressure force,  $d\mathbf{F}_p$ , on a differentially small area  $d\mathbf{A}$  is,

$$d\mathbf{F}_p = p(-d\mathbf{A}), \quad (1.78)$$

where  $p$  is the pressure acting on the surface. Notice that the area is a vector. Areas have both a magnitude and a direction. The area's direction is specified by the unit normal vector  $\hat{\mathbf{n}}$ , as shown in Figure 1.11. The pressure force acts inward on the area because the pressure force is the result of the momentum impulse of molecular impacts with the surface.

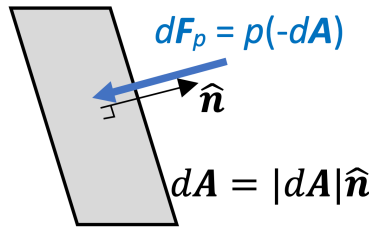


FIGURE 1.11. Illustration of the differentially small pressure force  $d\mathbf{F}_p$  acting inward on the differentially small area  $d\mathbf{A}$ .

- Absolute pressure is referenced to zero pressure. For example, a perfect vacuum has  $p_{\text{vacuum}} = 0$  (abs). To indicate an absolute pressure, one should use either the text “(abs)” after the reported pressure or, if using EE or BG units, specify the absolute pressure with the units “psia” or “psfa”.
  - There are no molecules in a perfect vacuum and, thus, there is no pressure.
  - Atmospheric pressure at standard conditions is  $p_{\text{atm}} = 101.33 \text{ kPa (abs)} = 1 \text{ atm (abs)} = 1.0133 \text{ bar (abs)} = 14.696 \text{ psia} = 760 \text{ torr (abs)}$ .
- Gage pressure is referenced to atmospheric pressure,

$$p_{\text{gage}} = p_{\text{abs}} - p_{\text{atm,abs}}. \quad (1.79)$$

- To indicate a gage pressure, one should use either the text “(gage)” after the reported pressure or, if using EE or BG units, specify the absolute pressure with the units “psig” or “psfg”.
- Atmospheric pressure at standard conditions is  $p_{\text{atm}} = 0$  (gage).
- A perfect vacuum has  $p_{\text{vacuum}} = -101.33 \text{ kPa (gage)} = -14.7 \text{ psig}$  (referencing to standard atmospheric conditions).
- An important equation of state for an ideal gas is the Ideal Gas Law,

$$p = \rho RT, \quad (1.80)$$

where  $p$  and  $T$  are absolute quantities, e.g.,  $[p] = \text{psia}$  or Pa (abs) and  $[T] = \text{°R}$  or K.

- The vapor pressure,  $p_v$ , of a liquid is the pressure at which the liquid is in equilibrium with its own vapor (Figure 1.12).

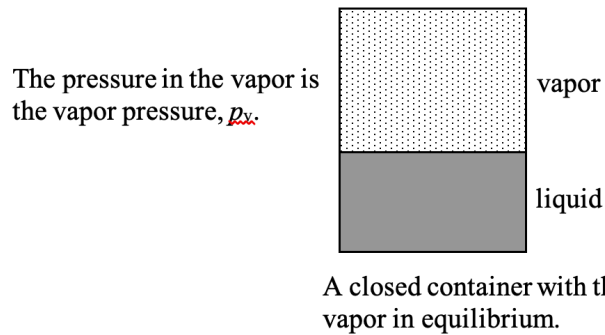


FIGURE 1.12. Illustration of liquid and vapor in equilibrium in a closed container. The pressure in the vapor is  $p_v$ .

- The vapor pressure for a liquid increases with increasing temperature.
- If the pressure in the liquid falls below the vapor pressure, then the liquid will turn to vapor. This change can occur via boiling or cavitation.
  - Boiling occurs when the temperature of the liquid increases so that the vapor pressure equals the surrounding atmospheric pressure. For example, the vapor pressure of water at 20 °C is 0.023 atm (abs) while the vapor pressure at 100 °C is 1 atm (abs). Hence, one method of turning liquid water to vapor is to bring the water temperature to 100 °C while holding the surrounding pressure at 1 atm (abs). This path is known as boiling and is shown schematically in the phase plot shown on the left-hand side of Figure 1.13.
  - Cavitation occurs when the surrounding pressure drops below the vapor pressure. Using the previous example, we can also turn liquid water to water vapor by dropping the surrounding pressure to 0.023 atm (abs) at 20 °C. This path is shown schematically in the phase plot shown on the right-hand side of Figure 1.13.

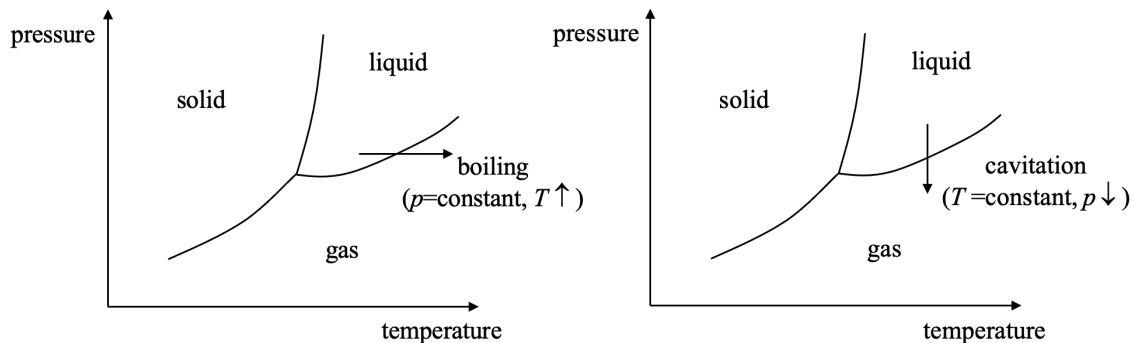


FIGURE 1.13. Phase diagrams (pressure vs. temperature) showing the processes of boiling (left) and cavitation (right).

- Cavitation can cause considerable damage to surfaces. When cavitation occurs in a liquid, pockets of vapor form (either as bubbles or large “voids”). When the vapor pockets travel into a region where the surrounding pressure is greater than the vapor pressure, the vapor region rapidly collapses. This collapse can be so rapid that shock waves and high speed water jets propagate from the collapsing region and impact on nearby surfaces causing small bits of the surface to erode away. Hence, cavitation is typically avoided when designing pumps, pipe bends, and underwater propellers.

*Example:*

A tire pressure gage measures the pressure inside a tire as 40 psig for a car at Standard Temperature and Pressure. What is the corresponding absolute pressure in the tire?

*Solution:*

The corresponding absolute pressure is,

$$p_{\text{tire,abs}} = p_{\text{tire,gage}} + p_{\text{atm,abs}} = 40 \text{ psi} + 14.7 \text{ psi} = 54.7 \text{ psia.} \quad (1.81)$$

Be sure to:

- Always indicate whether a pressure is an absolute or gage pressure in order to avoid ambiguity.
- Use an absolute pressure, and not a gage pressure, in the Ideal Gas Law.
- Be careful not to mix gage and absolute pressures when evaluating pressure forces.
- Use the correct area when calculating a pressure force.
- Integrate to find a pressure force when the pressure is not uniform over the area over which the pressure acts. This topic is covered in detail in Chapter 2.
- Check for cavitation in low pressure flows of liquids.

### 1.7.3. Temperature

- The dimensions of temperature are simply temperature  $\theta$ . Typical units are K, °R, °C, and °F.
- An object’s temperature is a quantitative way of describing how “hot” the object is (temperature is a measure of the agitation or random kinetic energy of the molecules). We typically measure an object’s temperature using a device called a “thermometer.”
- Experience tells us that when two objects are placed in contact with each other and have different temperatures, the hotter object (i.e., the one with a larger temperature) will become cooler while the cooler object becomes hotter. When the two objects have the same temperature, they no longer change temperature. The objects are then said to be in thermal equilibrium.
- A simple but fundamental concept concerning thermal equilibrium is the Zeroth Law of Thermodynamics: If two bodies are in thermal equilibrium with a third body, then the two bodies will also be in thermal equilibrium. This concept is key when comparing the temperatures of two objects not in contact using a thermometer since the thermometer acts as the third body.
- To use the concept of temperature, we must first define some scale on which we’ll measure temperatures. Perhaps the easiest scale to define is one where we reference all temperatures to some reproducible and unique physical phenomena such as the freezing or boiling points of water. Two point scales use two phenomena to define the temperature scale. For example, let’s define the ice point of water (when ice and water are in equilibrium at a pressure of 1 atm (abs)) as our 0° temperature and the steam point of water (when water and water vapor are in equilibrium at a pressure of 1 atm (abs)) as our 100° temperature. We can now measure all temperatures relative to this scale. Examples of two-point scales include the Celsius and Fahrenheit scales:

|                  | ice point of H <sub>2</sub> O | steam point of H <sub>2</sub> O |
|------------------|-------------------------------|---------------------------------|
| Celsius scale    | 0 °C                          | 100 °C                          |
| Fahrenheit scale | 32 °F                         | 212 °F                          |

Long ago, researchers noticed something curious when measuring the temperature of various gases at different pressures (and constant volume). They found that the temperature of a gas at a constant volume and at low pressures is proportional to its pressure, (Figure 1.14),

$$T = a + bp, \quad (1.82)$$

where  $a$  and  $b$  are constants. By extrapolating the temperature of the gases at zero pressure, we find that the lowest possible temperature, or the absolute zero temperature, is  $-273.15^\circ\text{C}$  on the Celsius scale and  $-459.67^\circ\text{F}$  on the Fahrenheit scale (i.e.,  $a = -273.15^\circ\text{C} = -459.67^\circ\text{F}$ ). This temperature scale, based on the behavior of ideal gases, is referred to as the ideal gas temperature scale.

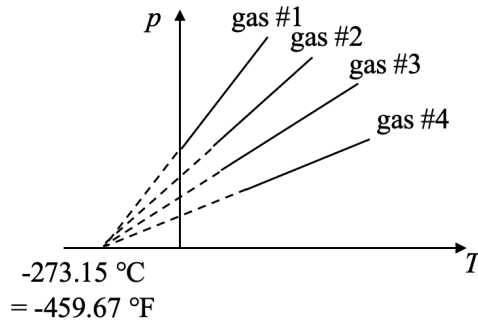


FIGURE 1.14. Gas pressure plotted as a function of temperature for different gases using a two-point temperature scale.

To make things a bit easier, let's redefine our scale so that absolute zero is the zero point of our temperature scale (Figure 1.15), i.e.,

$$T = bp \quad (a = 0). \quad (1.83)$$

This new scale is called an absolute temperature scale since the lowest temperature is zero.

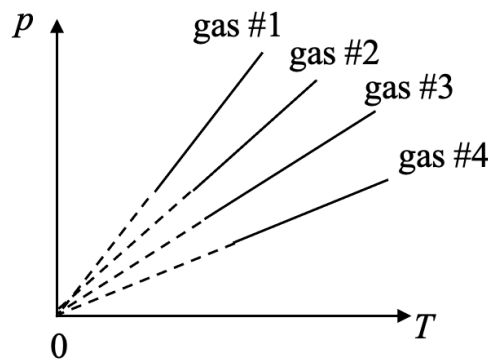


FIGURE 1.15. Gas pressure plotted as a function of temperature for different gases using an absolute temperature scale.

Note that so far our temperature scales have been based on the behavior of a particular substance (e.g., water) or a particular class of substances (e.g., gases). A better way to define a temperature scale is to make the scale independent of substances. Such a scale is called a thermodynamic temperature scale. In order to define such a temperature scale, we would need to first learn about the Second Law of Thermodynamics, a topic covered in Chapter 3. Suffice it to say here that the scale using this method gives the same result as the one using the ideal gas temperature scale.

To summarize, the lowest possible temperature is  $0 \text{ K} = 0 \text{ }^\circ\text{R}$  and  $\Delta(1 \text{ K}) = \Delta(1 \text{ }^\circ\text{C})$  and  $\Delta(1 \text{ }^\circ\text{R}) = \Delta(1 \text{ }^\circ\text{F})$ . Some additional helpful conversions are (in the equations below,  $\theta$  refers to temperature):

$$\theta(\text{K}) = 1.8\theta(^{\circ}\text{R}) \quad (1.8 = 9/5), \quad (1.84)$$

$$\theta(^{\circ}\text{C}) = [\theta(^{\circ}\text{F}) - 32]/1.8, \quad (1.85)$$

$$\theta(^{\circ}\text{C}) = \theta(\text{K}) - 273.15, \quad (1.86)$$

$$\theta(^{\circ}\text{F}) = \theta(^{\circ}\text{R}) - 459.67. \quad (1.87)$$

Another convenient conversion formula is,

$$10 \text{ }^{\circ}\text{C} = 50 \text{ }^{\circ}\text{F} \quad \text{and for every } 5 \text{ }^{\circ}\text{C increase, add } 9 \text{ }^{\circ}\text{F}. \quad (1.88)$$

Another very approximate conversion, and one that should only be used for everyday convenience and not in engineering calculations is,

$$\theta(^{\circ}\text{F}) \approx 2\theta(^{\circ}\text{C}) + 30 \quad (\text{a few degrees error over the range of typical weather temps}), \quad (1.89)$$

$$\theta(^{\circ}\text{C}) \approx [\theta(^{\circ}\text{F}) - 30]/2. \quad (1.90)$$

*Example:*

A pressure gage measures the pressure inside a car tire as 240 kPa (gage) at the National Institute of Standards and Technology (NIST) Standard Temperature and Pressure. To get the tire to a volume of 10 L, what mass of air needs to be added to the tire?

*Solution:*

The corresponding absolute pressure inside the tire is,  $p_{\text{tire,abs}} = 240 \text{ kPa} + 101.325 \text{ kPa} = 341.325 \text{ kPa}$  (abs). The absolute temperature inside the tire is  $T_{\text{tire,abs}} = 293.15 \text{ K}$ . From the Ideal Gas Law,

$$M = \frac{p_{\text{abs}}V}{R_{\text{air}}T_{\text{abs}}} = \frac{(341.325 \times 10^3 \text{ Pa})(10 \text{ L}) \left( \frac{1 \text{ m}^3}{1000 \text{ L}} \right)}{(287.058 \text{ J kg}^{-1} \text{ K}^{-1})(293.15 \text{ K})} = 4.06 \times 10^{-2} \text{ kg}, \quad (1.91)$$

where  $R_{\text{air}}$  is the gas constant for air.

Be sure to:

- (1) Use an absolute temperature when using the Ideal Gas Law.

#### 1.7.4. (Dynamic) Viscosity, $\mu$

- Viscosity is the “internal friction” within a fluid. It’s a measure of how easily a fluid flows.
- The dimensions of dynamic viscosity are  $FT/L^2$ . Typical units are Pa s and  $\text{lb}_f\text{s}/\text{ft}^2$ . Another common unit is the Poise (P) (pronounced “pwäz”):  $10 \text{ P} = 1 \text{ Pa s}$  and  $1 \text{ cP} = 0.01 \text{ P} = 1 \times 10^{-3} \text{ Pa s}$ .
- The viscous stresses in a fluid will be related to the deformation rate of a small element of fluid. Recall that for solids, the force is related to the deformation, e.g., Hooke’s Law for springs. For fluids, however, the forces are related to deformation rates.

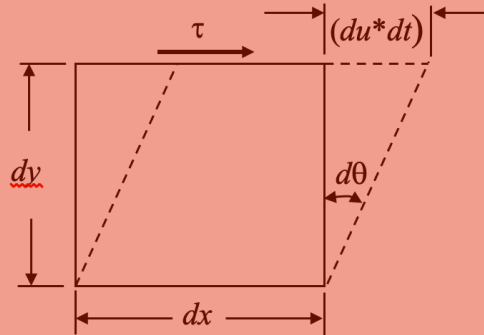


FIGURE 1.16. A small element of fluid subject to a small degree of shear deformation.

Consider the deformation of a small piece of fluid with area ( $dx dy$ ) as shown in Figure 1.16. The top of the fluid element is subject to a shear stress,  $\tau$ , over a short time,  $dt$ . During this time, the top of the fluid element moves with a small velocity,  $du$ , with respect to the bottom of the element. The total distance the top moves relative to the bottom will be  $du dt$ . The angular deformation of the fluid element can be measured by the angle the vertical sides of the element have deformed. The small angle  $d\theta$  is found from simple trigonometry,

$$\tan(d\theta) = \frac{du dt}{dy}. \quad (1.92)$$