

CHAPTER 1

The Basics
1.1. Symbolic vs. Numeric Analysis

Consider the following example. You need to determine the trajectory of a projectile fired from a cannon. The projectile has a mass of 10 kg and the cannon is tilted at an angle of 30° from the horizontal. The initial velocity of the projectile from the cannon is 100 m s^{-1} . Determine:

- (1) the distance the projectile will travel and
- (2) how long the projectile is in flight.

We can approach this problem a couple of different ways. The first is to start with the given numbers and immediately begin the calculations. The second approach is to solve the problem symbolically and then substitute the numbers at the end.

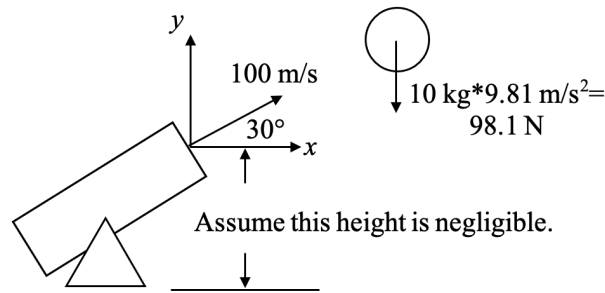
1.1.1. Numerical Approach


FIGURE 1.1. The free body diagram for the projectile example using numerical values.

Draw a free body diagram (FBD) of the projectile, as shown in Figure 1.1. Use Newton's Second Law to determine the acceleration of the projectile,

$$\sum F_x = m\ddot{x} \implies 0 \text{ N} = (10 \text{ kg})\ddot{x} \implies \ddot{x} = 0, \quad (1.1)$$

$$\sum F_y = m\ddot{y} \implies (10 \text{ kg})(-9.81 \text{ m/s}^2) = (10 \text{ kg})\ddot{y} \implies \ddot{y} = -9.81 \text{ m/s}^2. \quad (1.2)$$

Integrate with respect to time to determine the projectile's velocity and position given the projectile's initial x and y velocities and positions,

$$\dot{x} = \dot{x}_0 = (100 \text{ m s}^{-1})(\cos 30^\circ) = 86.6 \text{ m s}^{-1}, \quad (1.3)$$

$$\dot{y} = (-9.81 \text{ m/s}^2)t + \dot{y}_0 = (-9.81 \text{ m/s}^2)t + (100 \text{ m s}^{-1})(\sin 30^\circ) = (-9.81 \text{ m/s}^2)t + 50 \text{ m s}^{-1}, \quad (1.4)$$

$$\boxed{x = (86.6 \text{ m s}^{-1})t}, \quad (1.5)$$

$$\boxed{y = (-4.91 \text{ m/s}^2)t^2 + (50 \text{ m s}^{-1})t}. \quad (1.6)$$

The projectile will hit the ground when $y = 0$ so that by rearranging Eq. (1.6) we find that the time aloft is,

$$t = \frac{50.0 \text{ m s}^{-1}}{4.91 \text{ m/s}^2} = 10.2 \text{ s}. \quad (1.7)$$

Substituting into Eq. (1.5) gives the distance traveled as,

$$x = (86.6 \text{ m s}^{-1})(10.2 \text{ s}) = 883 \text{ m}. \quad (1.8)$$

As you can see, we've made a number of calculations along the way to finding the answers. Now let's address some additional questions based on these answers. How does the maximum time aloft depend on the mass of the projectile? If the initial speed from the cannon doubles, how is the range affected? What angle maximizes the distance the projectile travels? The answers to these questions are not obvious from Eqs. (1.5) – (1.8). We would need to perform additional calculations. Also, consider how many calculations would need to be made if we had to determine the range and time aloft for a variety of cannon angles, initial velocities, and cannon ball masses.

1.1.2. Symbolic Approach

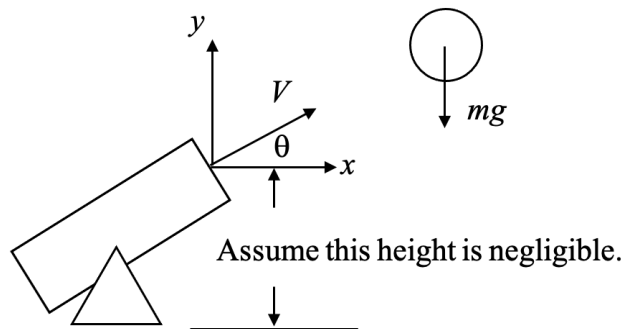


FIGURE 1.2. The free body diagram for the projectile example using symbols.

Now let's try working the same problem using symbols rather than numbers. We'll plug in the numbers at the very end of the problem. Draw the FBD as before (Figure 1.2). Follow the same approach as before,

$$\sum F_x = m\ddot{x} \implies 0 = m\ddot{x} \implies \ddot{x} = 0, \quad (1.9)$$

$$\sum F_y = m\ddot{y} \implies -mg = m\ddot{y} \implies \ddot{y} = -g, \quad (1.10)$$

$$\dot{x} = \dot{x}_0 = V \cos \theta, \quad (1.11)$$

$$\dot{y} = -gt + \dot{y}_0 = -gt + V \sin \theta, \quad (1.12)$$

$$x = (V \cos \theta)t \quad (x_0 = 0), \quad (1.13)$$

$$y = -\frac{1}{2}gt^2 + (V \sin \theta)t \quad (y_0 = 0). \quad (1.14)$$

The time aloft is found by setting $y = 0$,

$$t = \frac{2V \sin \theta}{g}, \quad (1.15)$$

and the distance traveled is,

$$x = \frac{2V^2 \cos \theta \sin \theta}{g} = \frac{V^2 \sin(2\theta)}{g}. \quad (1.16)$$

We can now plug in the given numbers to get our numerical answers,

$$t = \frac{2(100 \text{ m s}^{-1}) \sin(30^\circ)}{9.81 \text{ m/s}^2} = 10.2 \text{ s}, \quad (1.17)$$

$$x = \frac{(100 \text{ m s}^{-1})^2 \sin(60^\circ)}{9.81 \text{ m/s}^2} = 883 \text{ m}, \quad (1.18)$$

$$(1.19)$$

which are the same answers found previously.

Using these results for t and x we can easily calculate the time aloft and distance traveled for a variety of values of θ , V , and m . Note that nowhere in Eqs. (1.13) - (1.16) does the mass appear so we conclude that the mass of the cannon ball is unimportant to our calculations. We also observe that if we double the initial velocity, the time aloft will double and the distance traveled will quadruple. This information is easily lost in our calculations where numbers were used right away (refer to Eqs. (1.7) and (1.8)).

Lastly, if we wanted to determine the angle that will maximum the distance traveled for a given velocity, we observe from Eq. (1.16) that we want $\sin(2\theta)$ to be as large as possible. Thus, we should tilt our cannon at an angle of $\theta = 45^\circ$. Substituting this result back into Eqs. (1.15) and (1.16) gives,

$$t_{\max} = \frac{\sqrt{2}V}{g}, \quad (1.20)$$

$$x_{\max} = \frac{V^2}{g}. \quad (1.21)$$

We can also easily double-check the dimensions of the equations and verify that they are dimensionally homogeneous,

$$[t] = \frac{L/T}{L/T^2} = T \quad \text{Ok!} \quad (1.22)$$

$$[x] = \frac{(L/T)^2}{L/T^2} = L \quad \text{Ok!}, \quad (1.23)$$

where L and T represent length and time, respectively. We can conclude from this exercise the following:

- (1) More information is contained in our solutions when using the symbolic approach than when using the numeric approach.
- (2) If several calculations must be made using different values of the parameters, solving the problem first symbolically rather than starting the problem immediately with the numbers can save considerably on the number of computations required. Furthermore, it's much easier to correct numerical mistakes at the end of the problem rather than at the beginning or in the middle of the problem.

You're almost always better off working out a problem using symbols rather than numbers!

Be Sure To:

- (1) Work out problems symbolically and wait to substitute numerical values until the final relation has been derived.
- (2) Try to physically interpret your equations.
- (3) Make sure any relations you derive and the numbers you calculate are physically reasonable.
- (4) Double check that the dimensions (or units) of your answers are correct.

1.1.3. A Note on the Use of the Ballistic Equation

From your introductory physics course, you likely recall the ballistic equation for describing the position of an object, x , as a function of time, t , subject to an acceleration a , initial velocity, \dot{x}_0 , and initial position, x_0 ,

$$x = \frac{1}{2}at^2 + \dot{x}_0t + x_0. \quad (1.24)$$

This equation was derived in the following manner. Assume an object is subject to a constant acceleration a so we can write,

$$\ddot{x} = \frac{d\dot{x}}{dt} = a, \quad (1.25)$$

where the overdots represent differentiation with respect to time. Integrate this equation twice with respect to time making use of the initial conditions $x(t=0) = x_0$ and $\dot{x}(t=0) = \dot{x}_0$ to get,

$$\frac{d\dot{x}}{dt} = a \implies \int_{\dot{x}=\dot{x}_0}^{\dot{x}=\dot{x}} d\dot{x} = \int_{t=0}^{t=t} a dt \implies \dot{x} - \dot{x}_0 = a \int_{t=0}^{t=t} dt = at \implies \dot{x} = at + \dot{x}_0, \quad (1.26)$$

$$\dot{x} = \frac{dx}{dt} = at + \dot{x}_0 \implies \int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (at + \dot{x}_0) dt \implies x - x_0 = \frac{1}{2}at^2 + \dot{x}_0 t \implies x = \frac{1}{2}at^2 + \dot{x}_0 t + x_0. \quad (1.27)$$

A key step in the derivation to this equation is the assumption that $a = \text{constant}$. When a is a constant, it may be pulled out of the integrals in Eqs. (1.26) and (1.27). Thus, the ballistic equation is only valid when $a = \text{constant}$. It is not valid when a varies with time. If a is a function of time, then it must be evaluated within the integral. For example, if we have,

$$a = ct, \quad (1.28)$$

where c is a constant, then, using the same initial conditions as before, we have,

$$\frac{d\dot{x}}{dt} = a \implies \int_{\dot{x}=\dot{x}_0}^{\dot{x}=\dot{x}} d\dot{x} = \int_{t=0}^{t=t} a dt \implies \dot{x} - \dot{x}_0 = \int_{t=0}^{t=t} (ct) dt = \frac{1}{2}ct^2 \implies \dot{x} = \frac{1}{2}ct^2 + \dot{x}_0, \quad (1.29)$$

$$\dot{x} = \frac{dx}{dt} = \frac{1}{2}ct^2 + \dot{x}_0 \implies \int_{x=x_0}^{x=x} dx = \int_{t=0}^{t=t} (\frac{1}{2}ct^2 + \dot{x}_0) dt \implies x - x_0 = \frac{1}{6}ct^3 + \dot{x}_0 t \implies x = \frac{1}{6}ct^3 + \dot{x}_0 t + x_0. \quad (1.30)$$

Thus, we see that the position in Eq. (1.30) is different than the result given by the ballistic equation.

1.2. Dimensions and Units

A dimension is a qualitative description of the physical nature of some quantity.

Notes:

- (1) A basic or primary dimension is one that is not formed from a combination of other dimensions. It is an independent quantity.
- (2) A secondary dimension is one that is formed by combining primary dimensions.
- (3) Common dimensions include:

$M = \text{mass}$

$L = \text{length}$

$T = \text{time}$

$\theta = \text{temperature}$

$F = \text{force}$

- (4) If $M, L,$ and T are primary dimensions, then $F = ML/T^2$ is a secondary dimension. If $F, L,$ and T are primary dimensions, then $M = FT^2/L$ is a secondary dimension.

A unit is a quantitative description of a dimension. A unit gives “size” to a dimension. Common systems of units in engineering are given in Table 1.1.

Notes:

- (1) The mole is the amount of substance that contains the same number of elementary entities as there are atoms in 12 g of carbon 12 ($= 6.022 \times 10^{23}$, known as Avogadro’s constant). The elementary entities must be specified, e.g., atoms, molecules, particles, etc. The unit kmol (aka kgmol) is also frequently used, with $1 \text{ kmol} = 1000 \text{ mol} = 6.022 \times 10^{26}$ entities. The unit lbmol is used in the English system of units. Since $1 \text{ lbm} = 0.453 \text{ kg}$, $1 \text{ lbmol} = 453.592 \text{ mol}$. The number of kmols of a substance, n , is found by dividing the mass of the substance, m (kg) by the molecular weight of the substance, M (in kg/kmol): $n = m/M$. For example, the atomic weight of carbon 12 is 12

TABLE 1.1. Common units used in engineering.

primary dimension	SI (Système International d' Unités)	BG (British Gravitational)	EE (English Engineering)
L , length	meter (m)	foot (ft)	foot (ft)
T , time	second (s)	second (s)	second (s)
θ , temperature	Kelvin (K)	degree Rankine ($^{\circ}\text{R}$)	degree Rankine ($^{\circ}\text{R}$)
M , mass	kilogram (kg)	-not primary-	pound mass (lbm or lb)
N , amount of a substance	mole (mol)	mole (mol)	pound mole (lbmol)
F , force	-not primary-	pound force (lbf)	pound force (lbf)

kmol/kg, hence, 1 kg of carbon 12 contains: $(12 \text{ kg})/(12 \text{ kg/kmol}) = 1 \text{ kmol} = 1000 \text{ mol}$ (or 12 g of C12 contains 1 mol).

- (2) In the SI system, force is a secondary dimension and is given by: $F = ML/T^2$ where $1 \text{ N} = 1 \text{ kgm/s}^2$.
- (3) In the EE system, both force and mass are primary dimensions. The two are related via Newton's second law, $g_c F = ma$ where $g_c = 32.2 \text{ lb}_m\text{ft}/(\text{lb}_f\text{s}^2)$. It's easiest just to remember that: $1 \text{ lb}_f = 32.2 \text{ lb}_m\text{ft/s}^2$.
- (4) The slug is the unit of mass in the British Engineering system of units. To convert between a slug and lb_f is: $1 \text{ lb}_f = 1 \text{ slugft/s}^2$.
- (5) The kilogram-force (kgf) is (unfortunately) a not uncommon quantity. The conversion between kgf and Newtons is: $1 \text{ kgf} = 9.81 \text{ N}$.
- (6) It's a good policy to carry units through your calculations. Remember that, unless it's dimensionless, every number has a unit attached to it. For example, if $m = 1 \text{ lb}_m$ and $g = 32.2 \text{ ft/s}^2$,

Poor Practice: $mg = (1)(32.2) = 1 \text{ lb}_f$,

Good Practice: $mg = (1 \text{ lb}_m)(32.2 \text{ ft/s}^2) = (32.2 \text{ lb}_m\text{ft/s}^2) \left(\frac{1 \text{ lb}_f}{32.2 \text{ lb}_m\text{ft/s}^2} \right) = 1 \text{ lb}_f$

Carrying through your units makes it less likely that you'll make unit conversion errors, plus it makes it easier for you and others to follow your work.

Example: What is 70 mph in furlongs per fortnight?

Solution:

$$\left(70 \frac{\text{miles}}{\text{hour}} \right) \left(\frac{5280 \text{ feet}}{\text{miles}} \right) \left(\frac{\text{rod}}{16.5 \text{ feet}} \right) \left(\frac{\text{furlong}}{40 \text{ rods}} \right) \left(\frac{24 \text{ hours}}{\text{day}} \right) \left(\frac{7 \text{ days}}{\text{week}} \right) \left(\frac{2 \text{ weeks}}{\text{fortnight}} \right) = 1.88 \times 10^5 \frac{\text{furlongs}}{\text{fortnight}} \quad (1.31)$$

Example: What is weight of 1 lb_m of water in lb_f ?

Solution: Weight is mass multiplied by gravitational acceleration,

$$W = mg = (1 \text{ lb}_m) (32.2 \text{ ft/s}^2) \left(\frac{1 \text{ lb}_f}{32.2 \text{ lb}_m\text{ft/s}^2} \right) = 1 \text{ lb}_f. \quad (1.32)$$

Dimensional homogeneity is the concept whereby only quantities with similar dimensions can be added (or subtracted). It is essentially the concept of "You can't add apples and oranges." For example, consider the following equation,

$$10 \text{ kg} + 16 \text{ K} = 26 \text{ m s}^{-1}. \quad (1.33)$$

This equation doesn't make sense since it is not dimensionally homogeneous. How can one add mass to temperature and get velocity?!?

Note that dimensional homogeneity is a necessary, but not sufficient, condition for an equation to be correct. In other words, an equation must be dimensionally homogeneous to be correct, but a dimensionally homogeneous equation isn't always correct. For example,

$$10 \text{ kg} + 10 \text{ kg} = 25 \text{ kg}. \quad (1.34)$$

The equation has the right dimensions, but the wrong magnitudes!

Be sure to:

- (1) Verify that equations are dimensionally homogeneous.
- (2) Carefully evaluate unit conversions. A unit conversion error caused the loss of the \$125M Mars Climate Observer spacecraft in 1999!
- (3) Always include units with numerical values. An Air Canada Flight in 1983 (now referred to as the “Gimli Glider”) ran out of fuel and had to make an emergency landing due, in part, because the fuel load was assumed to be in pounds when in fact it was reported in kilograms.

The Ideal Gas Law is used to find the volume as given in the following formula,

$$V = \frac{mRT}{p},$$

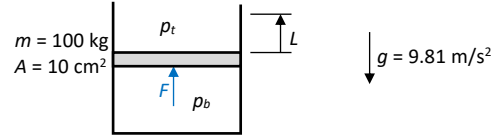
where $m = 2 \text{ kg}$, $R = 0.189 \text{ kJ}/(\text{kg}\cdot\text{K})$, $T = 300 \text{ K}$, and $p = 1 \text{ bar (abs)}$. Calculate the volume in m^3 . Show all of your calculations and unit conversions.

SOLUTION:

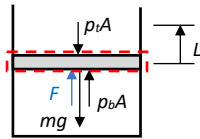
$$V = \frac{(2 \text{ kg})(0.189 \text{ kJ}/(\text{kg}\cdot\text{K}))(300 \text{ K})}{(1 \text{ bar})} = \left(\frac{2 \text{ kg}}{1}\right) \left(\frac{0.189 \text{ kJ}}{\text{kg}\cdot\text{K}}\right) \left(\frac{300 \text{ K}}{1}\right) \left(\frac{1}{1 \text{ bar}}\right) \left(\frac{1000 \text{ J}}{1 \text{ kJ}}\right) \left(\frac{1 \text{ bar}}{10^5 \text{ Pa}}\right) \left(\frac{1 \text{ Pa}}{1 \text{ N}/\text{m}^2}\right) \left(\frac{1 \text{ N}\cdot\text{m}}{1 \text{ J}}\right) \quad (1)$$

$$\boxed{V = 1.13 \text{ m}^3}$$

A piston, with a mass of $m = 100 \text{ kg}$ and cross-sectional area of $A = 10 \text{ cm}^2$, is located within a cylinder as shown in the figure. The pressure on the top surface of the piston is a uniform $p_t = 200 \text{ kPa}$ (abs). The pressure on the bottom of the piston is a uniform $p_b = 1 \text{ bar}$ (abs). If a force is applied to the piston to move it slowly upwards, i.e., in a quasi-equilibrium process, a distance of $L = 1 \text{ cm}$, determine the work done on the piston by the force, in kJ. Show all of your unit conversions.



SOLUTION:



First, determine the force by performing a vertical force balance on the piston, keeping in mind that the process is in quasi-equilibrium so that there is no acceleration of the piston,

$$\sum F = 0 = F + p_b A - mg - p_t A \Rightarrow F = mg + (p_t - p_b)A. \quad (1)$$

The work due to the force is,

$$W = \int_{s=0}^{s=L} \mathbf{F} \cdot d\mathbf{s} = \int_0^L [mg + (p_t - p_b)A] ds = [mg + (p_t - p_b)A]L, \quad (2)$$

$$\boxed{W = mgL + (p_t - p_b)AL}. \quad (3)$$

Note that the first term on the right-hand side is the change in potential energy while the second term is the pressure difference multiplied by the volume traced out by the piston ($= AL$).

Using the given values, evaluate the terms in Eq. (3). Start with the mgL term,

$$mgL = \underbrace{(100 \text{ kg})}_{=m} \underbrace{\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}_{=g} \underbrace{(1 \text{ cm})}_{=L} \left(\frac{1 \text{ N}}{1 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right) \left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \left(\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) = 9.81 * 10^{-3} \text{ kJ}. \quad (4)$$

Now evaluate the $(p_t - p_b)AL$ term,

$$(p_t - p_b)AL = \left[\underbrace{(200 \text{ kPa})}_{=p_t} \left(\frac{1000 \text{ Pa}}{1 \text{ kPa}}\right) - \underbrace{(1 \text{ bar})}_{=p_b} \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}}\right) \right] \underbrace{(10 \text{ cm}^2)}_{=A} \underbrace{(1 \text{ cm})}_{=L} \left(\frac{1 \text{ N/m}^2}{1 \text{ Pa}}\right) \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 \left(\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) = 1 * 10^{-3} \text{ kJ}. \quad (5)$$

Combining the numerical values in Eqs. (4) and (5),

$$\boxed{W = 1.08 * 10^{-2} \text{ kJ}}.$$

For the flow of gas in a nozzle,

$$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2),$$

where h_1 and h_2 are the gas's specific enthalpies at the inlet and outlet of the nozzle, respectively, and V_1 and V_2 are the gas speeds at the inlet and outlet, respectively. For the current case, $h_1 = 300$ kJ/kg, $V_1 = 100$ m/s, and $V_2 = 200$ m/s.

Using the given formula, calculate the value for h_2 in kJ/kg.

SOLUTION:

Substitute the given parameter into the equation to solve for h_2 , including appropriate unit conversions,

$$h_2 = \left(300 \frac{\text{kJ}}{\text{kg}}\right) + \frac{1}{2} \left[\left(100 \frac{\text{m}}{\text{s}}\right)^2 - \left(200 \frac{\text{m}}{\text{s}}\right)^2 \right] \left(\frac{1 \text{ kJ}}{1000 \text{ N}\cdot\text{m}}\right) \left(\frac{1 \text{ N}\cdot\text{m}}{1 \text{ kg}\cdot\text{m}^2/\text{s}^2}\right), \quad (1)$$

$$h_2 = \left(300 \frac{\text{kJ}}{\text{kg}}\right) + \frac{1}{2} \left(-30000 \text{ m}^2/\text{s}^2\right) \left(\frac{1 \text{ kJ}/\text{kg}}{1000 \text{ m}^2/\text{s}^2}\right), \quad (2)$$

$$h_2 = \left(300 \frac{\text{kJ}}{\text{kg}}\right) - \left(15 \frac{\text{kJ}}{\text{kg}}\right), \quad (3)$$

$$\boxed{h_2 = 285 \frac{\text{kJ}}{\text{kg}}}, \quad (4)$$

1.4. Significant Digits

Significant digits (aka significant figures) are the number of digits in a value that contribute to the accuracy of the value. A large number of significant digits implies that the value is known to high accuracy. For example, let's say we report the length of a mobile phone as 14.681230924 cm. Having such a large number of digits (in this case, 11 significant digits) implies that we know the phone length very accurately, in this case down to the nanometer scale. Obviously this isn't typically the case. A more reasonable length to report would be 14.7 cm (three significant digits), implying we know the length down to the millimeter. Thus, one should use an appropriate number significant digits when reporting values. Clearly the reporting of significant digits is closely related to the uncertainty in a reported value.

Following are the rules for identifying the number of significant digits in a value.

- (1) Leading zeros are not counted as significant digits. For example, 0.0023 has only two significant digits, the 2 and the 3. The three leading zeros do not count as significant digits and only serve as placeholders (the ones, tenths, and hundredths spots).
- (2) Trailing zeros that serve merely as placeholders are not counted as significant digits. For example, 23000 has only two significant digits, the 2 and the 3. The three zeros are not significant and only serve to fill in the ones, tens, and thousands spots.
- (3) Zeros between non-zero values are considered significant. For example, 23001 has five significant digits: 2, 3, 0, 0, and 1.
- (4) Trailing zeros immediately before and immediately after a decimal point are considered significant digits because they indicate that a value is known with some accuracy. For example, 23000. has five significant digits. Similarly, 2.3000 has five significant digits.
- (5) A line over a zero can indicate that the numbers to the left of that value are significant. For example, 230 $\bar{0}$ 0 has four significant digits: the 2, the 3, the 0 in the thousands spot, and the zero in the tens spot.
- (6) A handy way of determining the number of significant digits is to write the value in scientific notation then count the digits in the mantissa, i.e., the number being multiplied by 10 raised to some power. For example, 0.0023 can be written as 2.3×10^{-3} . The mantissa is 2.3, which has two significant digits. The number 23000 can be written as 2.3×10^4 , which has two significant digits. The numbers 23000. and 2.3000 can be written as 2.3000×10^4 and 2.3000×10^1 , respectively, and have five significant digits each.
- (7) An exact number has an infinite number of significant digits. For example, if the number of apples you have is five, then the number 5 has an infinite number of significant digits since it could be written as 5.0000... with an infinite number of zeros.

There are rules for performing calculations with significant figures so that calculated values are not reported more precisely than the numbers used in the calculations.

- (1) For addition and subtraction, the rightmost significant digit in the calculated value is determined by the least accurate added/subtracted value. For example, the sum $1.2 + 0.444$ should be reported as 1.6 since the accuracy of the sum is set by the tenths position of the first number. Similarly, $1.234 - 4.5$ should be reported as -3.3 , where the significant digit in the tenths position has been rounded.
- (2) For multiplication and division, the number of significant digits in the product/quotient should have the same number of significant digits as the lesser of the two multiplied/divided values. For example, 1.23×0.4 should be written as 0.5. The product has one significant digit and has been rounded. The operation $1.2/0.456$ should be written as 2.6.
- (3) There are also rules for logarithms, exponential, and trig functions, but these are not covered in these notes and are instead available in introductory physics textbooks or online.
- (4) To prevent a loss of accuracy, when performing intermediate calculations retain all of the digits as is practical rather than rounding the intermediate values. Only round the final result. For example, $(1.23 \times 0.4) + 5.67$ is calculated as $0.492 + 5.67$ which should then be reported as 6.2.

Despite all of these rules, many engineering analyses (including most of the examples I present in these notes) only follow these rules loosely. A best practice would be to follow the rules exactly. The author's only excuse for not following the rules is that it would be a considerable effort to correct all of the examples in these notes, which isn't a great excuse. An engineering rule of thumb is to present final values to within three significant digits, i.e., 1.23×10^n (where n is some number) since most engineering calculations don't involve highly accurate values. That being said, if one handles numbers and calculations requiring high accuracy, then following the significant digit rules carefully would be wise.

1.5. Experimental Uncertainty

In any experimental (or even computational) study, attention must be paid to the uncertainties involved in making measurements. Including the uncertainty allows one to judge the validity or accuracy of the measurements. Uncertainty analysis can also be useful when designing an experiment so that the propagation of uncertainties can be minimized. Consider a measurement of a flow rate through a pipe. Let's say that one measures a flow rate of 1.6 kg/s. Now consider a theoretical calculation that predicts a flow rate of 1.82 kg/s. Are the theory and measurement inconsistent? The answer depends upon the uncertainty in the measurement. If the experimental uncertainty is ± 0.3 kg/s, then the true measured value could very well be equal to the theoretical value. However, if the experimental uncertainty is ± 0.1 kg/s, then the two results are likely to be inconsistent.

There are two parts to uncertainty analysis. These include:

- (1) estimating the uncertainty associated with a measurement and
- (2) analyzing the propagation of uncertainty in subsequent analyses.

Both of these parts will be reviewed in the following sections. There are many texts (such as Holman, J.P., *Experimental Methods for Engineers*, McGraw-Hill) that can be referred to for additional information concerning experimental uncertainty.

1.5.1. Estimation of Uncertainty

There are three common types of error. These include "blunders," systematic (or fixed) errors, and random errors.

- (1) "Blunders" are errors caused by mistakes occurring due to inattention or an incorrectly configured experimental apparatus. Examples include:
 - Blatant blunder: An experimenter looks at the wrong gauge or misreads a scale and, as a result, records the wrong quantity.
 - Less blatant blunder: A measurement device has the wrong resolution (spatial or temporal) to measure the parameter of interest. For example, an experimenter who uses a manometer to measure the pressure fluctuations occurring in an automobile piston cylinder will not be able to capture the rapid changes in pressure due to the manometer's slow response time.
 - Subtle blunder: A measurement might affect the phenomenon that is being measured. For example, an experimenter using an ordinary thermometer to make a very precise measurement of a hot cavity's temperature might inadvertently affect the measurement by conducting heat out of the cavity through the thermometer's stem.
- (2) Systematic (or fixed) errors occur when repeated measurements are in error by the same amount. These errors can be removed via calibration or correction. For example, the error in length caused by a blunt ruler. This error can be corrected by calibrating the ruler against a known length.
- (3) Random errors occur due to unknown factors. These errors are not correctable, in general. Blunders and systematic errors can be avoided or corrected. It is the random errors that we must account for in uncertainty analyses. How we quantify random errors depends on whether we conduct a single experiment or multiple experiments. Each case is examined in the following sub-sections.

CHAPTER 3

Introductory Thermodynamics**3.1. Basics****3.1.1. What is thermodynamics?**

Thermodynamics is the study of energy, work, and heat, and the transformation between these quantities.

3.1.2. Where is thermodynamics used?

- power generation, e.g., fossil fuel power plants, internal combustion engines, gas turbine engines, solar thermal power plants
- high speed flows of gases, i.e., compressible flows, e.g., jet engines, rockets, high speed aircraft
- heating, ventilation, and air conditioning
- refrigeration
- combustion
- phase changes (evaporation, condensation, sublimation)

Thermodynamics serves as a foundation for many other topics, including fluid mechanics and heat transfer.

3.1.3. Definitions

- A closed system (aka system, aka control mass) is a particular quantity of matter chosen for study. The system may change shape and location, but it is always the same matter.
- The surroundings consist of everything that is not the system.
- The boundary of a system is the surface separating the system and surroundings.
- An isolated system is a closed system that does not interact with its surroundings. For example, if the system consists of air in a sealed, rigid, insulated container, then the air may be considered an isolated system since it has no mass, work, or heat transfer with the surroundings.
- A control volume (CV) (aka open system) is a particular volume chosen for study. Unlike a system, matter may change within a control volume. Note that the control volume does not need to remain fixed in size or location; it may move or change size and shape.
- A control surface (CS) is the surface enclosing a control volume. The orientation of the CS at a particular location is given by the direction of its outward-pointing unit normal vector, $\hat{\mathbf{n}}$, at that location. The outward-pointing unit normal vector has a magnitude of one, is perpendicular to the control surface, and always points out of the CV (Figure 3.1).
- Properties are macroscopic characteristics of a system. Example properties include mass, volume, energy, pressure, and temperature. A quantity is a property if and only if its change in value between two states is independent of the process between these states. For example, pressure is a property since its value only depends on the current state, but the work done on a system is not since the work depends on the process taken to reach a given state.
- An extensive property is one that depends on the mass in the system. For example, kinetic energy and mass are extensive properties since their values are proportional to the mass in the system.
- An intensive property is one that is independent of the mass in the system. For example, temperature and pressure are intensive properties since their values are independent of how much mass is in the system.

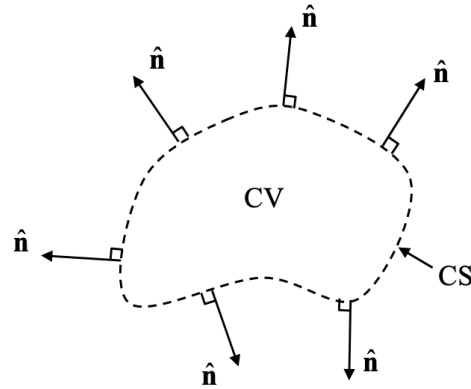


FIGURE 3.1. Illustration of control volume (CV), control surface (CS), and outward-pointing, unit normal vectors.

- A specific property is an extensive property per unit mass. A specific property is also an intensive property. An example of a specific property is specific volume $v = V/m$ where V is the system property and m is the system mass.
- An easy way to determine whether a property is extensive or intensive is to divide the system into two parts and see how the property is affected.
- The state of a system is the system's condition or configuration as described by its properties in sufficient detail so that it is distinguishable from other states. Often only a subset of properties is needed to define a state since some properties may be related.
- A process is the transformation of a system from one state to another. A few common processes include:
 - isothermal process: A process that occurs at constant temperature.
 - isobaric process: A process that occurs at constant pressure.
 - isochoric or isometric process: A process that occurs at constant volume.
 - adiabatic process: A process in which there is no heat transfer between the system and surroundings.
 - isentropic process: A process that occurs at constant entropy.
- A process is in steady state if the system's state does not change with time.
- A system is in a state of equilibrium if there are no potentials driving the system to another state. Examples of driving potentials include unbalanced forces, unbalanced temperatures, an electric potential (aka voltage).
- A process path is the series of states that a system passes through during some process.
- A quasi-equilibrium process is one where the process proceeds in such a manner that the system remains infinitesimally close to an equilibrium state at all times. One can interpret a quasi-equilibrium process as occurring slowly enough so that the system has time to adjust internally such that properties in part of the system do not change any faster than those properties in other parts of the system.
- A reversible process is one in which the system is in a state of equilibrium at all points in its path. In a reversible process, the system and the surroundings can be restored exactly to their initial states.
- An irreversible process is one where the system is not in a state of equilibrium at all points in its path. The system and surroundings cannot be returned to their exact initial states in an irreversible process. Note that all natural processes are irreversible. Several effects causing irreversibility include viscosity, heat conduction, and mass diffusion.

- A cycle is a sequence of processes that begins and ends at the same state. At the conclusion of a cycle, all properties have the same values they had at the beginning of the cycle. Thus, there is no change in the system's state at the end of a cycle.
- An equation of state is a relationship between properties of a particular substance or class of substances. Equations of state cannot be obtained from thermodynamics but are obtained either from experimental measurements or from some molecular model. Note that there can be various types of equations of state, e.g., two equations of state for an ideal gas include a thermal equation of state, which is the ideal gas law, $p = \rho RT$, and a caloric equation of state, which describes the relationship between the internal energy and temperature, $du = c_v dT$.

3.2. Energy, Work, and Heat

Now let's move our discussion to the three basic thermodynamic concepts of energy, work, and heat.

3.2.1. Energy

The energy associated with some phenomenon is not a physical quantity but is, in fact, just a number resulting from a formula containing physically measurable quantities related to that phenomenon. For example, the energy associated with the macroscopic motion of a system of mass, m , moving with a speed, V , is equal to $\frac{1}{2}mV^2$. By itself, the energy associated with a phenomenon is not a very useful quantity. However, experiments examining the total energy of a system, i.e., the sum of all the various energies, have resulted in a very remarkable observation. When the system does not interact with its surroundings, the total energy of the system remains constant. The energy associated with a particular phenomenon may change; however, it can only change at the expense of the energy associated with some other phenomenon. We'll examine this observation in greater detail a little later, but for now we will define the various types of energy that are most commonly encountered in engineering thermodynamics.

3.2.1.1. Kinetic Energy, KE

The energy associated with the macroscopic motion of a system relative to a coordinate system xyz is known as the kinetic energy, KE ,

$$KE = \frac{1}{2}mV^2, \quad (3.1)$$

where m is the mass of the system and V_{xyz} is the speed of the system in the coordinate system xyz .

3.2.1.2. Potential Energy, PE

The energy associated with a system's ability to do work in an external force field, such as a gravity field, is known as the potential energy, PE . For example, the gravitational potential energy for a mass, m , located in a gravitational field with gravitational acceleration, g , pointing in the $-z$ direction is,

$$PE = mgz, \quad (3.2)$$

where z is the height of the mass above some reference plane.

3.2.1.3. Internal Energy, U

The internal energy of a system is comprised of a number of sub-classes of energy which include:

- (1) sensible energy. This is the energy associated with the internal molecular translational, rotational, and vibrational motion. Temperature is a measure of this type of internal energy. The larger the temperature of a system, the greater its sensible energy.
- (2) latent energy. This is the energy associated with the attraction between molecules. We concern ourselves with latent energy most often when examining processes that involve a change of phase, such as going from a solid to a liquid or from a liquid to a gas (or vice versa).
- (3) chemical energy. This is the energy associated with the attraction between atoms.