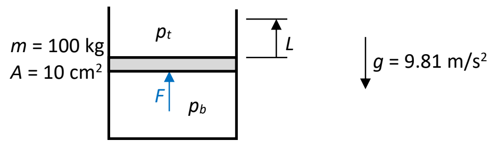
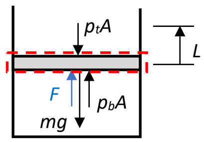
A piston, with a mass of m = 100 kg and cross-sectional area of A = 10 cm^2, is located within a cylinder as shown in the figure. The pressure on the top surface of the piston is a uniform p_t = 200 kPa (abs). The pressure on the bottom of the piston is a uniform p_b = 1 bar (abs). If a force is applied to the piston to move it slowly upwards, i.e., in a quasi-equilibrium process, a distance of L = 1 cm, determine the work done on the piston by the force, in kJ. Show all of your unit conversions.



SOLUTION:



First, determine the force by performing a vertical force balance on the piston, keeping in mind that the process is in quasi-equilibrium so that there is no acceleration of the piston,

$$\sum F = 0 = F + p_b A - mg - p_t A \;\Rightarrow\; F = mg + (p_t - p_b)A. \tag{1}$$

The work due to the force is,

$$W = \int_{s=0}^{s=L} \boldsymbol{F} \cdot d\boldsymbol{s} = \int_0^L [mg + (p_t - p_b)A]ds = [mg + (p_t - p_b)A]L, \tag{2}$$

$$\boxed{W = mgL + (p_t - p_b)AL}. \tag{3}$$

Note that the first term on the right-hand side is the change in potential energy while the second term is the pressure difference multiplied by the volume traced out by the piston (= *AL*).

Using the given values, evaluate the terms in Eq. (3). Start with the *mgL* term,

$$mgL = \underbrace{(100\ \text{kg})}_{=m}\underbrace{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}_{=g}\underbrace{(1\ \text{cm})}_{=L}\left(\frac{1\ \text{N}}{1\ \frac{\text{kg.m}}{\text{s}^2}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)\left(\frac{1\ \text{J}}{1\ \text{N.m}}\right)\left(\frac{1\ \text{kJ}}{1000\ \text{J}}\right) = 9.81 * 10^{-3}\ \text{kJ}. \tag{4}$$

Now evaluate the $(p_t - p_b)AL$ term,

$$(p_t - p_b)AL = \left[\underbrace{(200\ \text{kPa})}_{=p_t}\left(\frac{1000\ \text{Pa}}{1\ \text{kPa}}\right) - \underbrace{(1\ \text{bar})}_{=p_b}\left(\frac{10^5\ \text{Pa}}{1\ \text{bar}}\right)\right]\underbrace{(10\ \text{cm}^2)}_{=A}\underbrace{(1\ \text{cm})}_{=L}\left(\frac{1\ \text{N/m}^2}{1\ \text{Pa}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3\left(\frac{1\ \text{J}}{1\ \text{N.m}}\right)\left(\frac{1\ \text{kJ}}{1000\ \text{J}}\right) = 1 * 10^{-3}\ \text{kJ}. \tag{5}$$

Combining the numerical values in Eqs. (4) and (5),

$$\boxed{W = 1.08 * 10^{-2}\ \text{kJ}}.$$