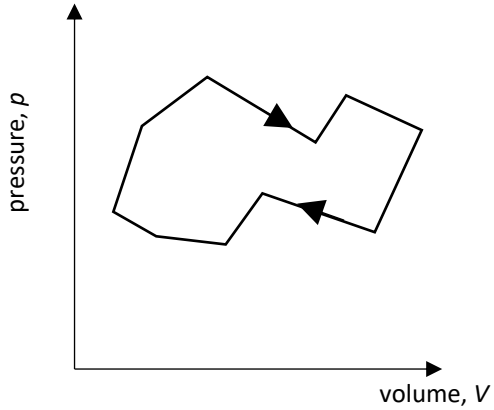
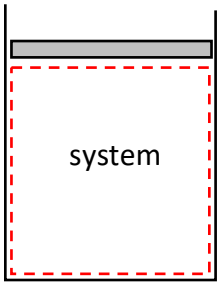
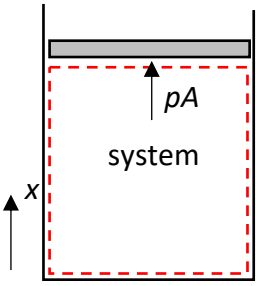
Show that for any clockwise-oriented cycle on a *p*-*V* plot, the net work done by the system will always be positive.





SOLUTION:

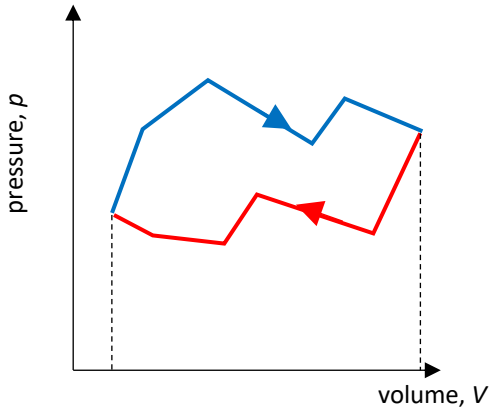


Recall that the work done by the system is positive when the system expands, i.e., the volume increases, and the work done by the system is negative when the system compresses, i.e., the volume decreases,

$$\delta W_{by\ sys} = \boldsymbol{F} \cdot d\boldsymbol{s} = pA\hat{\boldsymbol{\imath}} \cdot dx\hat{\boldsymbol{\imath}} = pAdx = pdV, \quad \text{(If } dV > 0\text{, then } \delta W_{by\ sys} > 0.\text{)} \tag{1}$$

$$W_{by\ sys} = \int_{V_1}^{V_2} pdV, \tag{2}$$

where p is the (absolute) pressure in the system and V is the system volume.



For an arbitrary, clockwise-oriented cycle, the expansion portion of the path (shown in blue in the previous figure) will have a larger average pressure than the compression portion of the path (shown in red in the figure). Hence, the magnitude of the work done by the system during expansion (area under the blue curve) will be larger than the magnitude of the work done by the system during compression (area under the red curve). In addition, $W_{by\ sys,expansion} > 0$ while $W_{by\ sys,compression} < 0$. Thus,

$$W_{by\ sys,cycle} = W_{by\ sys,expansion} + W_{by\ sys,compression} > 0 \quad \text{(for a clockwise oriented path).} \tag{3}$$

A similar argument may be made to show that,

$$W_{by\ sys,cycle} < 0 \quad \text{(for a counter-clockwise oriented path).} \tag{3}$$