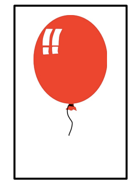

a. What is the minimum amount of work that must be done by the air in a balloon when it is inflated from a volume of 0.100 L to 1.85 L against a constant external pressure of 1.00 atm (abs)?

b. Now imagine that the balloon is inflated while in a sealed box, which has a volume of V_{box} = 0.010 m^3. How much work would be required to inflate the balloon between the same two volumes if the external absolute pressure varies as,

$$p_{ext} = \frac{p_0(V_{box} - V_0)}{V_{box} - V},$$

where p_0 = 1.00 atm (abs), V is the balloon volume, and V_0 = 0.100 L?

SOLUTION:
The system consists of the air in the balloon.

The work done by the system during the expansion process in part (a) is,

$$W_{by\,sys} = \int_{V_1}^{V_2} p dV, \tag{1}$$

$$W_{by\,sys} = p(V_2 - V_1). \quad \text{(Note that } p = \text{constant.)} \tag{2}$$

Here, p = 1.00 atm (abs) = 101 kPa (abs), V_1 = 0.100 L = 0.100*10^{-3} m^3, and V_2 = 1.85 L = 1.85*10^{-3} m^3. Thus,

$$W_{by\,sys} = (101 * 10^3 \text{ Pa})(1.85 * 10^{-3} \text{ m}^3 - 0.100 * 10^3 \text{ m}^3), \tag{3}$$

$$\boxed{W_{by\,sys} = 177 \text{ J.}} \tag{4}$$

If the external pressure changes with volume (part (b)), then,

$$W_{by\,sys} = \int_{V_1}^{V_2} p dV, \tag{5}$$

$$W_{by\,sys} = \int_{V_1}^{V_2} \frac{p_0(V_{box} - V_0)}{V_{box} - V} dV = -p_0(V_{box} - V_0)\ln\left(\frac{V_{box} - V_2}{V_{box} - V_1}\right), \tag{6}$$

$$W_{by\,sys} = -(101 * 10^3 \text{ Pa})(0.010 \text{ m}^3 - 0.100 * 10^{-3} \text{ m}^3)\ln\left(\frac{0.010 \text{ m}^3 - 1.85 * 10^{-3} \text{ m}^3}{0.010 \text{ m}^3 - 0.100 * 10^{-3} \text{ m}^3}\right), \tag{7}$$

$$\boxed{W_{by\,sys} = 194 \text{ J.}} \tag{8}$$

The work is larger for part (b) because the external pressure increases as the balloon expands. Note that as V_{box} increases in size, then the work for part (b) approaches the same value as that for part (a).