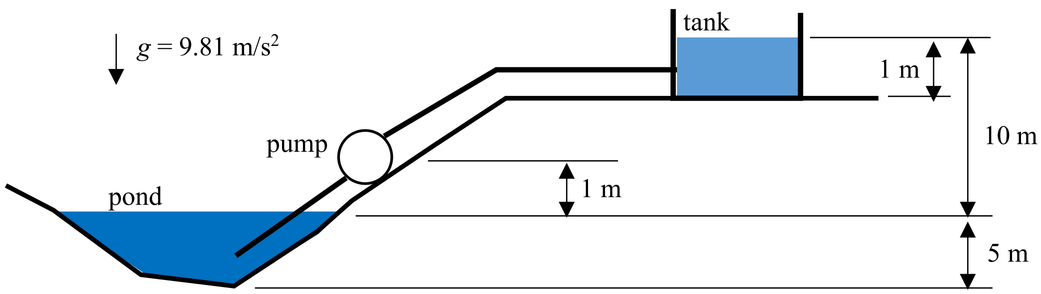
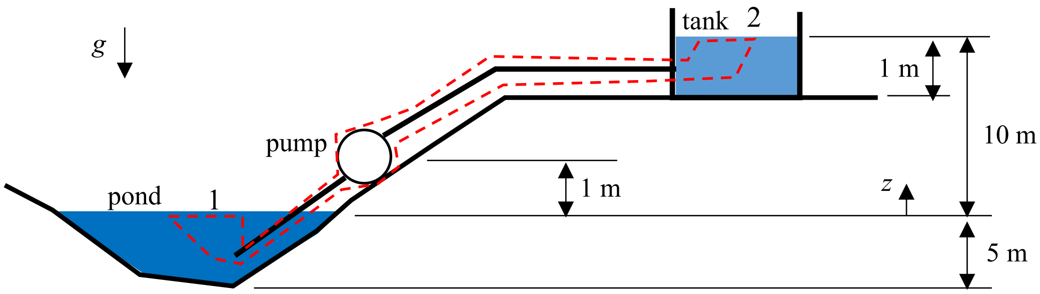
A homeowner wishes to design a pump and pipe system for watering her lawn. She connects the pipes and pump so that she can fill a tank from a nearby pond as shown in the following sketch. You may assume the system operates adiabatically.

a. Determine the minimum power per unit mass flow rate, in units of kJ/kg, required to operate the pump at steady state.
b. Prove that if irreversibilities are present, the temperature of the water in the tank will be larger than the water temperature in the pond.
c. Prove that if irreversibilities are present, the power per unit mass flow rate required to operate the pump will have a larger magnitude than the result found in part (a).



SOLUTION:



Apply Conservation of Mass to the control volume shown in the figure,

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \quad (1)$$

where,

$$\frac{dM_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (2)$$

$$\sum_{in} \dot{m} = \dot{m}_1, \quad (3)$$

$$\sum_{out} \dot{m} = \dot{m}_2, \quad (4)$$

Note that at the pond and tank surfaces the vertical speed of the water is small, but not zero. Thus, the mass flow rates at each surface won't be zero.

Substitute and simplify,

$$\dot{m}_1 = \dot{m}_1 = \dot{m}. \quad (5)$$

Now apply the 1st Law of Thermodynamics to the same control volume,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (6)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (7)$$

$$\dot{Q}_{into\ CV} = 0 \text{ (Assuming adiabatic operation.)}, \quad (8)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}[h_1 - h_2 + g(z_1 - z_2)]. \quad (9)$$

(The kinetic energies at the tank and pond surfaces are negligibly small.) (10)

Substitute and simplify,

$$\frac{\dot{W}_{by\ CV}}{\dot{m}} = h_1 - h_2 + g(z_1 - z_2). \quad (11)$$

Assume that water is an incompressible substance so that,

$$h_1 - h_2 = u_1 - u_2 - v(p_1 - p_2), \quad (12)$$

where v is a constant and $u = u(T)$. Note that at locations 1 and 2 the pressure is atmospheric, i.e.,

$$p_1 = p_2 = p_{atm}. \quad (13)$$

In order to determine the temperature change between states 1 and 2, apply the Entropy Equation to the same control volume,

$$\frac{dS_{CV}}{dt} = \int_b \frac{\delta\dot{Q}_{into\ CV}}{T} + \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \dot{\sigma}_{CV}, \quad (14)$$

where,

$$\frac{dS_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (15)$$

$$\int_b \frac{\delta\dot{Q}_{into\ CV}}{T} = 0 \text{ (Assuming adiabatic operation.)} \quad (16)$$

$$\sum_{in} \dot{m}s - \sum_{out} \dot{m}s = \dot{m}(s_1 - s_2). \quad (17)$$

Substituting and re-arranging gives,

$$s_2 - s_1 = \frac{\dot{\sigma}_{CV}}{\dot{m}}. \tag{18}$$

Again, assuming water is an incompressible substance,

$$s_2 - s_1 = \int_{T_1}^{T_2} c(T)\frac{dT}{T}. \tag{19}$$

Since we want the minimum power required to operate the pump, assume the process is internally reversible, which means that,

$$\dot{\sigma}_{CV} = 0. \tag{20}$$

Combining Eqs. (18), (19), and (20),

$$T_2 = T_1. \tag{21}$$

Since the temperature doesn't change, the specific internal energy doesn't change, i.e., $u_2 = u_1$. Hence, from Eq. (12) (and Eq. (13)),

$$h_2 = h_1.$$

Finally then, from Eq. (11),

$$\boxed{\frac{\dot{W}_{by\ CV,min}}{\dot{m}} = g(z_1 - z_2)}. \tag{22}$$

Using the given parameters,

$g = 9.81\ \text{m/s}^2$,

$z_1 = 0$,

$z_2 = 10$ m,

=> $\boxed{\frac{\dot{W}_{by\ CV,min}}{\dot{m}} = -0.0981\ \text{kJ/kg}}$ i.e., 0.0981 kJ/kg must go into the pump for the assumed conditions.

If internal irreversibilities are present, then,

$$\dot{\sigma}_{CV} > 0. \tag{23}$$

From Eq. (18), $s_2 > s_1$, and from Eq. (19), $T_2 > T_1$ (since $c(T) > 0$). Furthermore, if $T_2 > T_1$, then $u_2 > u_1$ and $h_2 > h_1$ (Eq. (12)). Thus, from Eq. (11),

$$\left|\frac{\dot{W}_{by\ CV,irrev}}{\dot{m}}\right| > \left|\frac{\dot{W}_{by\ CV,min}}{\dot{m}}\right|. \tag{24}$$