Air enters a one inlet/one outlet control volume operating at steady state. The inlet pressure and temperature are 100 psia and 900 ºR, respectively. Flow through the control volume is adiabatic and the outlet pressure is 25 psia. Kinetic and potentially energy changes across the control volume are negligible. Determine the rate of entropy production per unit mass flow rate for:

a. if the control volume encloses a turbine having an isentropic turbine efficiency of 89.1%, and
b. if the control volume encloses a throttling valve.

SOLUTION:

Apply the Entropy Equation to the control volume,

$$\frac{dS_{CV}}{dt} = \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \int_b \frac{\delta \dot{Q}_{in}}{T} + \dot{\sigma}, \tag{1}$$

where,

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \tag{2}$$

$$\sum_{in} \dot{m}s - \sum_{out} \dot{m}s = \dot{m}(s_1 - s_2), \tag{3}$$

(from COM, $\dot{m}_2 = \dot{m}_1 = \dot{m}$)

$$\int_b \frac{\delta \dot{Q}_{in}}{T} = 0 \quad \text{(assuming adiabatic operation)}, \tag{4}$$

$$\dot{\sigma} = ?, \tag{5}$$

Substitute and re-arrange to solve for the rate of entropy generation per unit mass flow rate,

$$0 = \dot{m}(s_1 - s_2) + \dot{\sigma}, \tag{6}$$

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1. \tag{7}$$

Assuming air is an ideal gas,

$$s_2 - s_1 = s^0(T_2) - s^0(T_1) - Rln\left(\frac{p_2}{p_1}\right), \tag{8}$$

Note that the inlet pressure and temperature are given, as well as the outlet pressure. To determine the outlet temperature for the turbine, first make use of the isentropic turbine efficiency to find the outlet specific enthalpy,

$$\eta_{turb,isen} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow \eta_{turb,isen}(h_1 - h_{2s}) = h_1 - h_2 \Rightarrow h_2 = h_1 - \eta_{turb,isen}(h_1 - h_{2s}). \tag{9}$$

If we can find h_2, then we can determine the corresponding T_2 from the Ideal Gas Table for air.

The value for h_1 may be found from the Ideal Gas Table for air,

$h_1 = h(T_1 = 900\ ^\circ\text{R}) = 216.26$ Btu/lbm.

To find h_{2s}, first find T_{2s} using Eq. (8) and noting that $s_{2s} = s_1$,

$$0 = s^0(T_{2s}) - s^0(T_1) - Rln\left(\frac{p_{2s}}{p_1}\right) \Rightarrow s^0(T_{2s}) = s^0(T_1) + Rln\left(\frac{p_{2s}}{p_1}\right) \tag{10}$$

Using the given data,

$p_1 = 100$ psia,

$p_{2s} = p_2 = 25$ psia,

$R = 0.06856$ Btu/(lbm.°R),

$s^0(T_1 = 900\ ^\circ\text{R}) = 0.72438$ Btu/(lbm.°R) (from the Ideal Gas Table),

⇨ $s^0(T_{2s}) = 0.62934$ Btu/(lbm.°R) => $T_{2s} = 608.29$ °R => $h_{2s} = 145.46$ Btu/lbm (from the Ideal Gas Table)

⇨ $h_2 = 153.18$ Btu/lbm => $T_2 = 640.37$ °R => $s^0(T_2) = 0.641729$ Btu/(lbm.°R).

Finally, substituting into Eq. (8) and Eq. (7),

$$\boxed{\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = 0.012393 \text{ Btu/(lb}_\text{m}.^\circ\text{R)}}.$$

Alternately, we could have found T_{2s} using the relative pressures since we have an isentropic process involving an ideal gas,

$$\frac{p_r(T_{2s})}{p_r(T_1)} = \frac{p_2}{p_1} \Rightarrow p_r(T_{2s}) = p_r(T_1)\left(\frac{p_2}{p_1}\right), \tag{11}$$

where,

$p_r(T_1 = 900\ ^\circ\text{R}) = 8.411$ (using the Ideal Gas Table),

$p_2 = 25$ psia,

$p_1 = 100$ psia,

⇨ $p_r(T_{2s}) = 2.10275$ => $T_{2s} = 608.01$ °R (using the Ideal Gas Table),

which is the same temperature at state 2*s* found previously, within numerical error.

Now assume a throttling valve is contained within the control volume. Equations (7) and (8) still hold for this scenario. To find the downstream temperature, apply the 1st Law to the control volume to obtain,

$$h_2 = h_1, \tag{12}$$

assuming steady state, negligible KE and PE changes, adiabatic flow, and no work. Recall that for an ideal gas, $h = h(T)$ and, thus, $T_2 = T_1$. Simplifying Eq. (8) gives,

$$s_2 - s_1 = -R ln\left(\frac{p_2}{p_1}\right), \tag{8}$$

Using the given data,

$$s_2 - s_1 = 0.095044 \text{ Btu/(lb}_\text{m}\text{.}^\text{o}\text{R)},$$

and, from Eq. (7),

$$\boxed{\frac{\dot{\sigma}}{\dot{m}} = 0.095044 \text{ Btu/(lb}_\text{m}\text{.}^\text{o}\text{R)}}.$$

Comparing the results for the turbine and throttling valves, we see that the turbine introduces considerably less irreversibility into the flow than the valve. The flow expansion through the turbine is more controlled (often done in stages rather than all at once), which is why the turbine has less irreversibility.