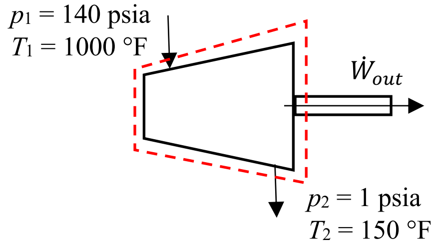
Water vapor at 1000 °F and 140 psia enters a turbine operating at steady state and expands to 1 psia and 150 °F. Stray heat transfer and kinetic and potential energy effects are negligible.

a. Determine the turbine power per unit mass flow rate.
b. What is the rate of entropy generation per unit mass flow rate?
c. Determine the maximum theoretical turbine power per unit mass flow rate for the same inlet state and exit pressure. What is the outlet temperature for these conditions?
d. Sketch the two processes on a *T*-*s* diagram.

Image: https://www.explainthatstuff.com/steam-turbines.html
Video: https://www.youtube.com/watch?v=SPg7hOxFItI

SOLUTION:



Apply the 1st Law to a CV surrounding the turbine,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out}, \quad (2)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (3)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_1 - h_2) \quad \text{(neglecting the KE and PE)}, \quad (4)$$

(Note that COM has been used, assuming steady state operation, to give $\dot{m} = \dot{m}_2 = \dot{m}_3$.)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \quad (5)$$

$$\dot{W}_{out} = ? \quad (6)$$

Substitute and solve for the power per unit mass flow rate,

$$0 = \dot{m}(h_1 - h_2) - \dot{W}_{out}, \quad (7)$$

$$\frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2. \quad (8)$$

Determine the specific enthalpies from the property tables for water,

State 1: p_1 = 140 psia (abs), T_1 = 1000 °F => (SHV) h_1 = 1531.0 Btu/lbm, s_1 = 1.8827 Btu/(lbm.°R),
State 2: p_2 = 1 psia (abs), T_2 = 150 °F => (SHV) h_2 = 1127.5 Btu/lbm. s_2 = 2.0151 Btu/(lbm.°R),

Substituting these values into Eq. (8) gives,

$$\boxed{\frac{\dot{W}_{out}}{\dot{m}} = 404 \text{ Btu/lb}_m}.$$

The rate of entropy production is found by applying the Entropy Equation to the same CV,

$$\frac{dS_{CV}}{dt} = \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \int_b \frac{\dot{Q}_{in}}{T} + \dot{\sigma}, \quad (9)$$

where,

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (10)$$

$$\sum_{in} \dot{m}s - \sum_{out} \dot{m}s = \dot{m}(s_1 - s_2), \quad (11)$$

$$\int_b \frac{\dot{Q}_{in}}{T} = 0 \quad \text{(assuming adiabatic operation)}. \quad (12)$$

$$\dot{\sigma} = ? \quad (13)$$

Substitute and solve for the rate of entropy production per unit mass flow rate,

$$0 = \dot{m}(s_1 - s_2) + \dot{\sigma}, \quad (14)$$

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1. \quad (15)$$

Using the previously determine specific entropy values,

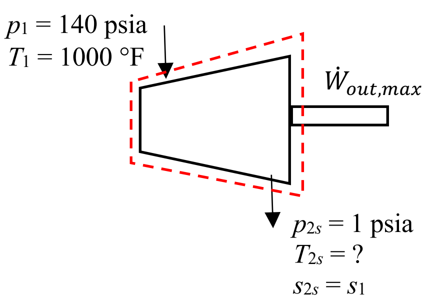
$$\boxed{\frac{\dot{\sigma}}{\dot{m}} = 0.1324 \text{ Btu/(lb}_\text{m}.°\text{R)}}.$$

Thus we observe that the actual flow through the turbine has internal irreversibilities.

To determine the maximum power out of the turbine, apply the 1st Law to the same CV, but this time assume that the flow through the turbine is adiabatic and internally reversible ($\dot{\sigma}$ = 0). An adiabatic and internally reversible flow is also isentropic. Thus,

$$s_{2s} = s_1. \quad (16)$$

Note that the subscript "2*s*" is used to indicate that state 2 now corresponds to an isentropic process.



Applying the 1st Law to this CV gives,

$$\frac{\dot{W}_{out,max}}{\dot{m}} = h_1 - h_{2s}. \quad (17)$$

State 2*s* can be found from the property tables:

State 2*s*: p_{2s} = 1 psia, $s_{2s} = s_1$ = 1.8827 Btu/(lbm.°R)

⇨ (SLVM) $\boxed{T_{2s} = 101.70\ °\text{F}}$, $s_{f,2s}$ = 0.1327 Btu/lbm. $s_{g,2s}$ = 1.9779 Btu/lbm,

⇨ $x_{2s} = \frac{s_{2s} - s_{f,2s}}{s_{g,2s} - s_{f,2s}} = 0.94841$,

⇨ $h_{2s} = (1 - x_{2s})h_{f,2s} + x_{2s}h_{g,2s}$ = 1052.35 Btu/lbm,

where $h_{f,2s}$ = 69.74 Btu/lbm and $h_{g,2s}$ = 1105.8 Btu/lbm.

Using this value for h_{2s} and the previous value for h_1, Eq. (10) gives,

$$\boxed{\frac{\dot{W}_{out,max}}{\dot{m}} = 479 \text{ Btu/lb}_\text{m}}.$$

Note that the maximum power outlet temperature is smaller than the temperature for the actual case (T_{2s} = 101.70 °F < T_2 = 150 °F). For the maximum power case, there is more efficient conversion of enthalpy (also referred to as "flow energy") into shaft work. Thus, the outlet enthalpy, and corresponding temperature, is smaller.

We can define a "turbine isentropic efficiency" as the actual power divided by the maximum possible power,

$$\eta_{isen,turbine} \equiv \frac{\dot{W}_{out,actual}}{\dot{W}_{out,isentropic}}. \tag{18}$$

Using the previously calculated values, the isentropic efficiency for this turbine is, $\eta_{\text{isen,turbine}} = 0.843 = 84.3\%$.