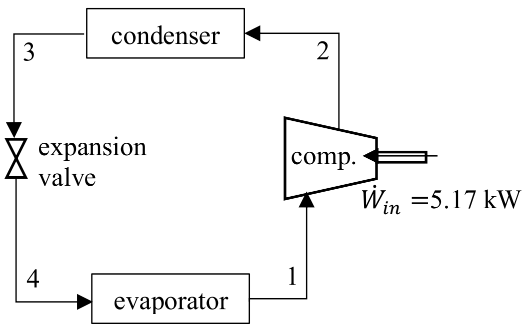
Consider the heat pump system shown below using R134a as the working fluid. The state conditions at various points in the system are also provided.



State 1:
$\dot{m}$ = 7 kg/min
p_1 = 2.4 bar (abs)

State 2:
p_2 = 9 bar (abs)
T_2 = 60 °C

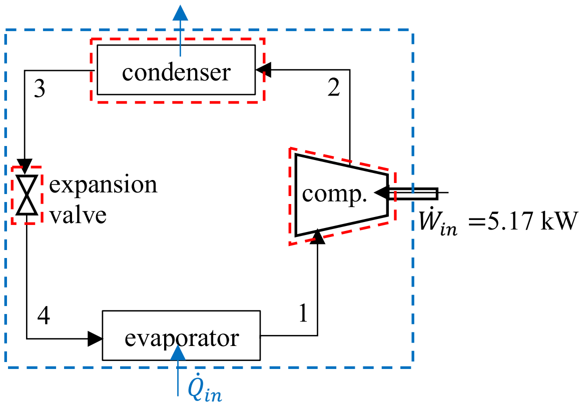
State 3:
saturated liquid
p_3 = 9 bar (abs)

State 4:
p_4 = 2.4 bar (abs)

a. Determine the coefficient of performance for this heat pump.
b. If the valve were replaced by a turbine, power could be produced, thereby reducing the power requirement of the heat pump system. Would you recommend this power-saving measure? Explain.

$\dot{Q}_{out}$

SOLUTION:



A heat pump's coefficient of performance, in terms of the variables used in the schematic (and using the blue CV shown), is,

$$COP_{HP} = \frac{\dot{Q}_{out}}{\dot{W}_{in}}. \tag{1}$$

In this problem the work into the compressor is given, but the heat transfer to the hot reservoir (from the condenser) is not given. To determine this heat transfer, apply the 1st Law to a CV surrounding the condenser,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) - \dot{Q}_{out} - \dot{W}_{out}, \tag{2}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \tag{3}$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_2 - h_3) \quad \text{(neglecting the KE and PE)}, \tag{4}$$

(Note that COM has been used, assuming steady state operation, to give $\dot{m} = \dot{m}_2 = \dot{m}_3$.)

$$\dot{Q}_{out} = ? \quad \text{(assuming adiabatic operation)}, \tag{5}$$

$$\dot{W}_{out} = 0 \quad \text{(the condenser is a passive device)}. \tag{6}$$

Substitute and solve for the rate of heat transfer,

$$0 = \dot{m}(h_2 - h_3) - \dot{Q}_{out}, \tag{7}$$

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3). \tag{8}$$

A similar analysis can be performed for the expansion valve to give,

$$h_4 = h_3. \tag{9}$$

Similarly, for the compressor,

$$\dot{W}_{in} = \dot{m}(h_2 - h_1), \tag{10}$$

$$h_1 = h_2 - \frac{\dot{W}_{in}}{\dot{m}}. \tag{11}$$

Use the given state information and the property tables to determine the specific enthalpies.

State 2: p_2 = 9 bar (abs), T_2 = 60 °C => (SHV) h_2 = 295.13 kJ/kg,
State 3: saturated liquid, p_3 = 9 bar (abs) => h_3 = 101.64 kJ/kg,
State 4: p_4 = 2.4 bar (abs), $h_4 = h_3$ = 101.64 kJ/kg (Eq. (9)),
State 1: p_1 = 2.4 bar (abs) => (SHV) h_1 = 250.8157 kJ/kg (Eq. (11), note that $\dot{m}$ =7 kg/min = 0.1167 kg/s).

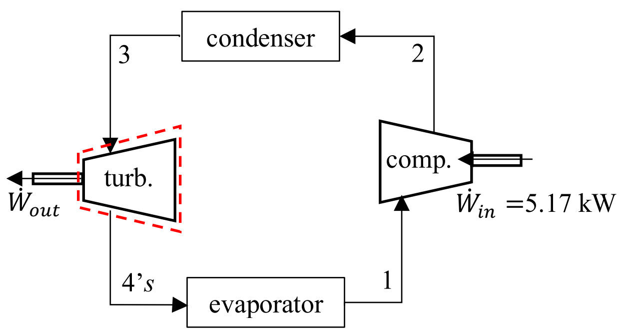
Using these data, Eq. (8) gives,

$\dot{Q}_{out}$ = 22.574 kW.

Substituting the given input power and the calculated heat transfer from the condenser in the coefficient of performance (Eq. (1)),

$\boxed{COP_{HP} = 4.37}$.

Now consider replacing the expansion valve with a turbine, as shown in the following figure. Note that state 4 is now labeled as state 4' since the conditions there may be different than the conditions for the expansion valve.



Applying the 1st Law to a CV surrounding the turbine gives,

$$\dot{W}_{out,s} = \dot{m}(h_3 - h_{4's}). \tag{12}$$

To determine the specific enthalpy at state 4's, assume the turbine operates at 100% isentropic efficiency, i.e., assume the turbine operates adiabatically and in an internally reversible manner, i.e., in an isentropic manner. We'll call this state 4's to reflect that it corresponds to an isentropic assumption. This operating conditions is chosen in order to determine the maximum power that can be generated by the turbine in order to evaluate a "best case scenario" for incorporating a turbine into the cycle. Since the flow through the turbine is assumed isentropic,

$s_{4's} = s_3 = s_{f3}$ = 0.37387 kJ/(kg.K) (from the SLVM-pressure table for a saturated liquid at p_3 = 9 bar). (13)

The pressure at state 4's is still the same as the pressure at state 1, i.e., $p_{4's} = p_1$ = 2.4 bar (abs), since flow through the evaporator is assumed to be isobaric. Using the SLVM-pressure table to determine the specific enthalpy at 4's,

$p_{4's}$ = 2.4 bar (abs), $s_{4's}$ = 0.37387 kJ/(kg.K) => (SLVM) $s_{f4's}$ = 0.17803 kJ/(kg.K) and $s_{g4's}$ = 0.93465 kJ/(kg.K),

$$\Rightarrow \quad x_{4's} = \frac{s_{4's} - s_{4f}}{s_{4g} - s_{4f}} = 0.258845, \tag{14}$$

The specific enthalpy at state 4's is then,

$$h_{4's} = (1 - x_{4's})h_{f4's} + x_{4's}h_{g4's} = 97.12966 \text{ kJ/kg}, \tag{15}$$

where $h_{f4's}$ = 44.686 kJ/kg and $h_{g4's}$ = 247.3 kJ/kg. Substituting this value back into Eq. (12) gives,

$\dot{W}_{out,s}$ =0.526 kW.

This is the maximum amount of power that can be generated by a turbine that is located where the expansion valve used to be.

Using the net power required to calculate the heat pump's coefficient of performance,

$$\dot{W}_{in,net} = \dot{W}_{in} - \dot{W}_{out,s} = 5.17\text{ kW} - 0.526\text{ kW} = 4.64\text{ kW}, \tag{16}$$

$$COP_{HP'} = \frac{\dot{Q}_{out}}{\dot{W}_{in,net}} = 4.86. \tag{17}$$

Thus, the coefficient of performance increases by approximately 11% [= ($COP_{HP'}$ – COP_{HP})/COP_{HP}*100%)] compared to the expansion valve case, assuming the turbine is operating at its most efficient condition. This increase in thermal efficiency, especially in non-ideal conditions, would likely not be worth the additional cost and complexity of incorporating a turbine and associated hardware into the heat pump. In addition, the refrigerant passing through the turbine would have a low quality (recall that $x_3 = 0$ and $x_{4's} = 0.256$), which would put significant wear on the turbine components. Turbines typically operate at qualities near one or in the superheated vapor region where liquid droplets impacting high speed turbine blades is less of a concern.