A 180 ft^3 tank initially filled with air at 1 atm (abs) and 70 °F is evacuated by a vacuum pump. During the process, the tank air is maintained at 70 °F. The vacuum pump discharges air to the surroundings at the surrounding's temperature and pressure, which are 70 °F and 1 atm (abs), respectively. Determine the minimum amount of work required to completely evacuate the tank, in Btu. Note that at the end of the discharge pipe downstream of the pump, the pressure is the same as the surroundings.

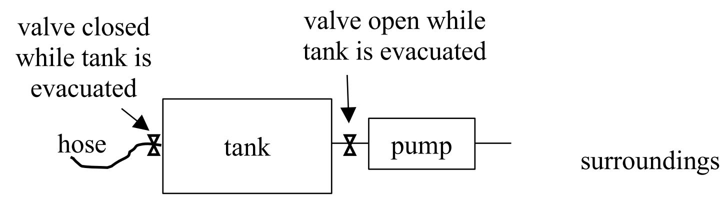
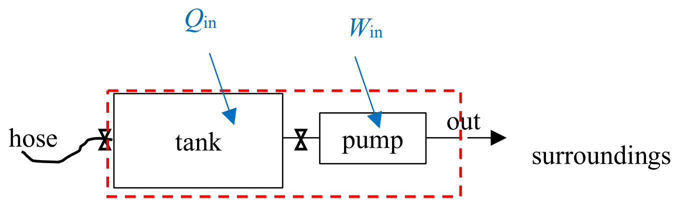


Image from: https://www.kwipped.com/rentals/power-utility/vacuum-trucktrailer/867

SOLUTION:



To determine the minimum amount of work required to evacuate the tank, apply the 1st Law to a control volume surrounding the tank and pump,

$$\Delta E = \sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) + Q_{in} + W_{in}, \quad (1)$$

where,

$$\Delta E = \Delta U + \Delta KE + \Delta PE = \Delta U = m_f u_f - m_i u_i = -m_i u_i, \quad (2)$$

(neglecting changes in the KE and PE within the CV; the tank is completely evacuated at the final state),

$$\sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) = -m_{out} h_{out}, \quad (3)$$

(since there is only mass leaving the CV; neglecting the KE and PE at the outlet),

$$Q_{in} = ?, \quad (4)$$

$$W_{in} = ?. \quad (5)$$

Substitute and solve for the work,

$$-m_i u_i = -m_{out} h_{out} + Q_{in} + W_{in}, \quad (6)$$

$$W_{in} = -m_i u_i + m_{out} h_{out} - Q_{in}. \quad (7)$$

Now apply Conservation of Mass to the same control volume,

$$\Delta M = \sum_{in} m - \sum_{out} m, \quad (8)$$

where,

$$\Delta M = m_f - m_i = -m_i \quad \text{(the tank is completely evacuated at the final state)}, \quad (9)$$

$$\sum_{in} m - \sum_{out} m = m_{in} - m_{out} = -m_{out}, \quad \text{(since there is only mass leaving the CV)}. \quad (10)$$

Substituting,

$$-m_i = -m_{out} \quad \Rightarrow \quad m_{out} = m_i. \quad (11)$$

Substitute Eq. (11) into Eq. (7) and simplify,

$$W_{in} = -m_i(u_i - h_{out}) - Q_{in}, \quad (12)$$

$$W_{in} = m_i(h_{out} - u_i) - Q_{in}. \quad (13)$$

To determine the heat transfer, apply the Entropy Equation to the same CV,

$$\Delta S = \sum_{in} ms - \sum_{out} ms + \int_b \frac{Q_{in}}{T} + \sigma, \quad (14)$$

where,

$$\Delta S = m_f s_f - m_i s_i = -m_i s_i \quad \text{(since the tank is evacuated at the final state)}, \quad (15)$$

$$\sum_{in} ms - \sum_{out} ms = m_{in} s_{in} - m_{out} s_{out} = -m_{out} s_{out} \quad \text{(since mass only leaves the CV)}, \quad (16)$$

$$\int_b \frac{Q_{in}}{T} = \frac{Q_{in}}{T_{surr}}, \quad (17)$$

(Note that since the surroundings and tank have the same temperature throughout the process, the temperature where the heat transfer is occurring is well known. If the tank temperature was different from the surroundings, then the CV would need to extend outside the thermal boundary layer adjacent to the tank to where the temperature remains at the surrounding temperature. For this case, the CV would include some of the surroundings.)

$$\sigma = ?. \quad (18)$$

Substitute and solve for the heat transfer, making use of Eq. (13),

$$-m_i s_i = -m_{out} s_{out} + \frac{Q_{in}}{T_{surr}} + \sigma, \quad (19)$$

$$Q_{in} = T_{surr}[m_i(s_{out} - s_i) - \sigma]. \quad (20)$$

Substitute Eq. (20) into Eq. (13),

$$W_{in} = m_i(h_{out} - u_i) - T_{surr} m_i (s_{out} - s_i) + T_{surr}\sigma. \quad (21)$$

Note that the minimum work occurs when the process is internally reversible, i.e., when $\sigma = 0$. For this case, Eq. (21) is,

$$W_{in,min} = m_i[h_{out} - u_i - T_{surr}(s_{out} - s_i)]. \quad (22)$$

Because the process occurs isothermally, $T_{out} = T_i$. Furthermore, at the exit of the pump discharge pipe, i.e., at the "out" location, the pressure is the same as the surrounding pressure, which happens to be the same initial pressure in the tank, i.e., $p_{out} = p_{surr} = p_i = 1$ atm (abs).

The mass initially in the tank may be found using the ideal gas law,

$$m_i = \frac{p_i V}{R T_i}. \quad (23)$$

Using the given data,

$p_i = 1$ atm (abs) = 2116.22 lb_f/ft^2,
$V = 180\ ft^3$,
$R_{air} = 53.3533\ (ft.lb_f)/(lb_m.°R)$,
$T_i = 70\ °F = 529.67\ °R$,
⇨ $m_i = 13.479\ lb_m$.

Now evaluate the change in specific entropy and the heat transfer,

$$s_{out} - s_i = s^0(T_{out}) - s^0(T_i) - R\ln\left(\frac{p_{out}}{p_i}\right) = 0, \quad \text{(treating air as an ideal gas; } T_{out} = T_i \text{ and } p_{out} = p_i\text{)}, \quad (24)$$

⇨ $Q_{in,\text{min work}} = 0$.

Lastly, evaluate the specific enthalpy and specific internal energy terms,

$h_{out} = h(T_{out} = 529.67\ °R) = 126.585\ Btu/lb_m$ (interpolating from the ideal gas tables),
$u_i = u(T_i = 529.67\ °R) = 90.2753\ Btu/lb_m$ (from the ideal gas tables),
⇨ $\boxed{W_{in,min} = 489\ Btu}$.

Alternately, we could have written the specific enthalpy as,

$$h_{out} = u_{out} + (pv)_{out}, \tag{25}$$

so that Eq. (22) becomes,

$$W_{in,min} = m_i[u_{out} + (pv)_{out} - u_i] \quad \text{(making use of Eq. (24))}, \tag{26}$$

$$W_{in,min} = m_i(u_{out} - u_i) + p_{out}V, \quad \text{(since } m_{out} = m_i \text{ and } V = m_{out}v_{out}\text{)}. \tag{27}$$

Since the process is isothermal, $T_{out} = T_i$ and the air is treated as an ideal gas ($u = u(T)$), $u_{out} = u_i$ and Eq. (27) becomes,

$$W_{in,min} = p_{out}V. \tag{28}$$

Using the given data,

p_{out} = 1 atm (abs) = 2116.22 lb_f/ft^2,

V = 180 ft^3,

⇨ $W_{in,min}$ = 38100 $ft.lb_f$ = 490 Btu, which is the same answer found previously, to within numerical error.