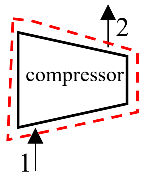
Air expands isothermally at steady state with no internal irreversibilities through a turbine from 10 bar (abs) and 500 K to 2 bar (abs). Determine the rate of heat transfer per unit mass flow rate of air and power per unit mass flow rate of air.

SOLUTION:



Apply the Entropy Equation to a control volume surrounding the turbine,

$$\frac{dS_{CV}}{dt} = \int_{CS} \frac{\delta \dot{Q}_{into\ CV}}{T} + \dot{\sigma}_{CV} + \sum_{in} \dot{m}s - \sum_{out} \dot{m}s, \tag{1}$$

where,

$$\frac{dS_{CV}}{dt} = 0, \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS} \frac{\delta \dot{Q}_{into\ CV}}{T} = \frac{\dot{Q}_{into\ CV}}{T}, \quad \text{(the process is isothermal, so the temperature doesn't vary around the CS)} \tag{3}$$

$$\dot{\sigma}_{CV} = 0, \quad \text{(an internally reversible process)} \tag{4}$$

$$\sum_{in} \dot{m}s - \sum_{out} \dot{m}s = \dot{m}(s_1 - s_2), \quad \text{(one inlet/one outlet and steady flow)} \tag{5}$$

Substitute and solve for the rate of heat transfer per unit mass flow rate,

$$0 = \frac{\dot{Q}_{into\ CV}}{T} + \dot{m}(s_1 - s_2), \tag{6}$$

$$\frac{\dot{Q}_{into\ CV}}{\dot{m}} = T(s_2 - s_1). \tag{7}$$

The change in the specific entropy, assuming ideal gas behavior, is,

$$s_2 - s_1 = s^0(T_2) - s^0(T_1) - R_{air} ln\left(\frac{p_2}{p_1}\right). \tag{8}$$

Note that since the process is isothermal, $T_2 = T_1$ and the previous equation may be simplified to,

$$s_2 - s_1 = -R_{air} ln\left(\frac{p_2}{p_1}\right). \tag{9}$$

Using the given data,

$T = 500$ K,
$R_{air} = 0.287$ kJ/(kg.K),
$p_1 = 10$ bar (abs),
$p_2 = 2$ bar (abs),
⇨ $s_2 - s_1 = 0.46191$ kJ/(kg.K),
⇨ $\boxed{\frac{\dot{Q}_{into\ CV}}{\dot{m}} = 231 \text{ kJ/kg}}$.

The power per unit mass flow rate may be found using the 1st Law applied to the same control volume,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right), \tag{10}$$

where,

$$\frac{dE_{CV}}{dt} = 0, \quad \text{(steady)} \tag{11}$$

$$\sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) = \dot{m}(h_1 - h_2), \tag{12}$$

(assuming changes in kinetic and potential energy are negligible)

Note that since for an ideal gas $h = h(T)$ and the process is isothermal, $h_2 = h_1$. Thus, simplifying Eq. (10) gives,

$$\boxed{\frac{\dot{W}_{by\ CV}}{\dot{m}} = \frac{\dot{Q}_{into\ CV}}{\dot{m}}}. \tag{13}$$