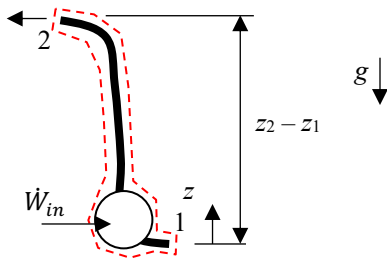


A 3 hp pump operating at steady state draws in liquid water at 1 atm (abs), 60 °F and delivers it at 5 atm (abs) at an elevation 20 ft above the inlet. There is no significant change in velocity between the inlet and exit. Is it possible to pump 1000 gal in 10 min or less? Explain.



Image: <https://www.bobvila.com/articles/some-advice-about-sump-pumps/>

SOLUTION:



The mass flow rate required to pump 1000 gal of liquid water in 10 min is,

$$\dot{m} = \rho Q, \quad (1)$$

where ρ is the density of liquid water, assumed here to be $62.4 \text{ lb}_m/\text{ft}^3$, and Q is the volumetric flow rate,

$$Q = (1000 \text{ gal})/(10 \text{ min}) = 100 \text{ gal/min} = 13.37 \text{ ft}^3/\text{min} = 0.223 \text{ ft}^3/\text{s} \quad (2)$$

$$\Rightarrow \dot{m} = 13.9 \text{ lb}_m/\text{s}.$$

Since we're interested in knowing if the pump is capable of pumping at the given flow rate, consider the ideal case, i.e., assume internally reversible, adiabatic flow. Note that the flow is at steady state and has one inlet and outlet. For these conditions, the 1st Law may be written as,

$$\frac{\dot{W}_{\text{other, on CV}}}{\dot{m}} = \int_{p_1}^{p_2} v dp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1). \quad (3)$$

For the current situation, assume the liquid water is incompressible (also re-write the specific volume as $v = 1/\rho$). Furthermore, we're told that there's no significant change in the velocity between the inlet and outlet, so the change in kinetic energy may be neglected. Re-writing Eq. (3) for these conditions gives,

$$\frac{\dot{W}_{\text{other, on CV}}}{\dot{m}} = \frac{P_2 - P_1}{\rho} + g(z_2 - z_1), \quad (4)$$

$$\dot{m} = \frac{\dot{W}_{\text{other, on CV}}}{\frac{P_2 - P_1}{\rho} + g(z_2 - z_1)}. \quad (5)$$

Using the given parameters,

$$\dot{W}_{\text{other, on CV}} = 3 \text{ hp} = 1650 \text{ ft}\cdot\text{lb}_f/\text{s},$$

$$p_1 = 1 \text{ atm (abs)} = 2117 \text{ lb}_f/\text{ft}^2, \quad p_2 = 5 \text{ atm (abs)} = 10580 \text{ lb}_f/\text{in}^2,$$

$$\rho = 62.4 \text{ lb}_m/\text{ft}^3,$$

$$g = 32.2 \text{ ft/s}^2, \quad z_2 - z_1 = 20 \text{ ft},$$

$$\Rightarrow \dot{m} = 10.6 \text{ lb}_m/\text{s}.$$

Since the ideal mass flow rate is smaller than what is required, it's not possible to pump the water at the desired flow rate.