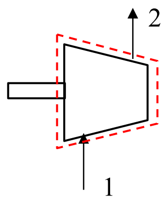
Nitrogen (N_2) enters an insulated compressor operating at steady state at 1 bar (abs) and 37 °C with a mass flow rate of 1000 kg/h and exits at 10 bar (abs). Kinetic and potential energy changes through the compressor are negligible. The nitrogen can be modeled as an ideal gas with a specific heat ratio of 1.391 and a specific heat at constant pressure of 1.056 kJ/(kg.K).

a. Determine the minimum theoretical power input required to operate the compressor and the corresponding exit temperature.
b. If the exit temperature is 397 °C, determine the power input and the compressor efficiency.

SOLUTION:

To find the power required to operate the compressor, apply the First Law to a control volume surrounding the compressor as shown in the following figure.



$$\frac{dE_{CV}}{dt} = \sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,on}\\ \text{CV}}}, \quad (1)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(steady flow assumed)}, \quad (2)$$

$$\sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \dot{m}_1 h_1 - \dot{m}_2 h_2 = \dot{m}\left(h_1 - h_2\right), \quad (3)$$

(changes in KE and PE neglected; from COM, the mass flow rates are the same)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(adiabatic operation since insulated).} \quad (4)$$

Solving for the power added into the compressor,

$$\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \dot{m}\left(h_2 - h_1\right). \quad (5)$$

If we further assume that the nitrogen behaves as a perfect gas, i.e., it has constant specific heats, which is a reasonable assumption if the temperature change is only a few hundred degrees, then,

$$\boxed{\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \dot{m}c_p\left(T_2 - T_1\right)}. \quad (6)$$

To calculate the <u>minimum</u> power required to operate the compressor, assume reversible operation. Since the flow is then adiabatic and reversible, it will also be isentropic. For isentropic operation of a perfect gas,

$$\frac{p_2}{p_1} = \left(\frac{T_{2s}}{T_1}\right)^{\frac{k}{k-1}} \Rightarrow T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}, \quad (7)$$

where the subscript "*s*" has been added to the temperature at state 2 to indicate isentropic conditions. Using the given parameters,

T_1 = 37 °C = 310 K,
p_2 = 10 bar (abs),
p_1 = 1 bar (abs),
k = 1.391,
$\Rightarrow T_{2s}$ = 592 K (= 319 °C)

Substituting into Eq. (6) gives,

$$\boxed{\left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\text{min}} = 82.8\text{ kW}},$$

with c_p = 1.056 kJ/(kg.K).

Using the actual measured temperature of T_2 = 397 °C = 670 K,

$$\boxed{\dot{W}_{\substack{\text{other,into}\\ \text{CV}}} = 106\text{ kW}.}$$

The efficiency of the compressor is given by,

$$\boxed{\eta = \frac{\left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\text{min}}}{\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}} = 0.78.}$$

If we assume ideal, rather than perfect, gas behavior, then outlet temperature corresponding to an isentropic process is found using,

$$s_2 - s_1 = 0 = s_2^0\left(T_{2s}\right) - s_1^0\left(T_1\right) - R\ln\left(\frac{p_2}{p_1}\right) \Rightarrow \bar{s}_2^0\left(T_{2s}\right) = \bar{s}_1^0\left(T_1\right) + \bar{R}_u\ln\left(\frac{p_2}{p_1}\right), \tag{8}$$

with,

$\bar{s}_1^0$ (T_1 = 310 K) = 192.638 kJ/(kmol.K) (Table A-23 in Moran et al., 8th ed.),

p_2/p_1 = (10 bar)/(1 bar) = 10,

$\bar{R}_u$ = 8.314 kJ/(kmol.K),

=> $\bar{s}_2^0$ = 211.78 kJ/(kmol.K) => $\boxed{T_{2s} = 594\text{ K}}$ (interpolating in Table A-23).

This result is less than 1% different from the one found earlier assuming perfect gas behavior.