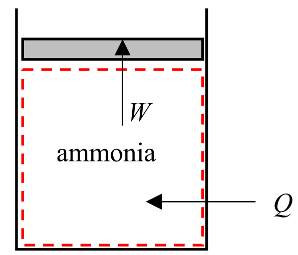
A mass of 0.5 kg of ammonia is contained in a piston-cylinder assembly, initially at -20 °C and a quality of 25%. The ammonia is slowly heated to a final state where the temperature is 20 °C and the pressure is 0.6 MPa (abs). The pressure varies linearly with specific volume during this heating process. There are no significant changes in kinetic or potential energy. For the ammonia,

a. show the process on a *p-v* sketch,
b. evaluate the work done by the ammonia, and
c. evaluate the heat transfer into the ammonia.

SOLUTION:
Let the ammonia be the system of interest.



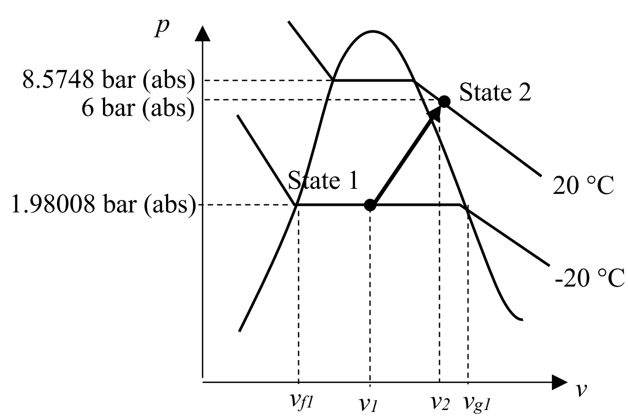
The given properties at State 1 are T_1 = -20 °C and x_1 = 0.25. Since a quality is given, the ammonia must be in a saturated state. Using the Saturated Ammonia table organized according to temperature, at T = -20 °C, p_{sat} = 1.9008 bar (abs). In addition, u_{f1} = 89.095 kJ/kg, u_{g1} = 1299.9 kJ/kg, v_{f1} = 0.0015035 m^3/kg, and v_{g1} = 0.62373 m^3/kg. Using these data, we can calculate the specific internal energy and specific volume at state 1,

$$u_1 = (1 - x_1)u_{f1} + x_1 u_{g1}, \tag{1}$$

$$v_1 = (1 - x_1)v_{f1} + x_1 v_{g1}, \tag{2}$$

⇨ u_1 = 391.796 kJ/kg, v_1 = 0.157060 m^3/kg.

At state 2, T_2 = 20 °C and p_2 = 0.6 MPa (abs) = 6 bar (abs). From the Saturated Ammonia table organized according to temperature, at T = 20 °C, p_{sat} = 8.5748 bar (abs). Since the saturation pressure is larger than p_2, the ammonia at state 2 must be a superheated vapor. Using the Superheated Vapor table for ammonia at the given temperature and pressure, u_2 = 1348.8 kJ/kg and v_2 = 0.22151 m^3/kg. These states are plotted on the following p-v diagram.



Note that the path from State 1 to State 2 is given in the problem statement as being linear.

The work done by the ammonia is pressure work and is given by,

$$W_{by\ sys,press} = \int_{V_1}^{V_2} p\,dV = \left(\frac{p_1 + p_2}{2}\right) m(v_2 - v_1), \tag{3}$$

where the work done by the ammonia may be found geometrically from the figure. Using the given parameters, including the ammonia mass of m = 0.5 kg,

$\boxed{W_{by\ sys,press} = 12.7\text{ kJ}}$.

The heat transfer into the system may be found using the 1st Law applied to the system,

$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}\ , \tag{4}$$

where,

$$\Delta E_{sys} = \Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} \approx \Delta U_{sys} \quad \text{(the } \Delta KE_{sys} \text{ and } \Delta PE_{sys} \text{ are assumed} \ll \Delta U_{sys}\text{).} \tag{5}$$

$$\Delta U_{sys} = m(u_2 - u_1), \tag{6}$$

Substituting the parameters previously calculated,

ΔU_{sys} = 478.50 kJ,
$\boxed{Q_{into\ sys} = 491.2\text{ kJ}}$

Note that the work done by the ammonia could also have been determined from evaluation of the integral,

$$W_{by\ sys,press} = \int_{V_1}^{V_2} pdV = m\int_{v_1}^{v_2} pdv, \tag{7}$$

where,

$$p = p_1 + \left(\frac{p_2-p_1}{v_2-v_1}\right)(v - v_1), \tag{8}$$

since it's given that the pressure is linearly related to the specific volume. Substituting Eq. (8) into Eq. (7) and evaluating,

$$W_{by\ sys,press} = m\int_{v_1}^{v_2}\left[p_1 + \left(\frac{p_2-p_1}{v_2-v_1}\right)(v - v_1)\right]dv = m\int_{v_1}^{v_2}\left[p_1 dv + \left(\frac{p_2-p_1}{v_2-v_1}\right)vdv - \left(\frac{p_2-p_1}{v_2-v_1}\right)v_1 dv\right], \tag{9}$$

$$W_{by\ sys,press} = m\left[p_1(v_2 - v_1) + \left(\frac{p_2-p_1}{v_2-v_1}\right)\frac{1}{2}(v_2^2 - v_1^2) - \left(\frac{p_2-p_1}{v_2-v_1}\right)v_1(v_2 - v_1)\right], \tag{10}$$

$$W_{by\ sys,press} = m\left[p_1(v_2 - v_1) + (p_2 - p_1)\frac{1}{2}(v_2 + v_1) - (p_2 - p_1)v_1\right], \tag{11}$$

$$W_{by\ sys,press} = m\left[\frac{1}{2}p_1 v_2 - \frac{1}{2}p_1 v_1 + \frac{1}{2}p_2 v_2 - \frac{1}{2}p_2 v_1\right], \tag{12}$$

$$W_{by\ sys,press} = m\frac{1}{2}(p_1 + p_2)(v_2 - v_1), \tag{13}$$

which is the same result as Eq. (3).