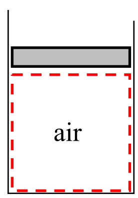
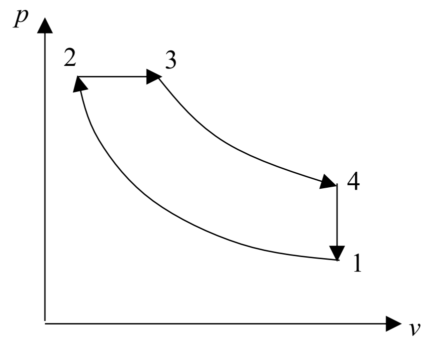
The displacement volume of an internal combustion engine is 3 L. The processes within each cylinder of the engine are modeled as an air-standard Diesel cycle with a cutoff ratio of 2.5. The state of the air at the beginning of compression is fixed by $p_1$ = 95 kPa (abs), $T_1$ = 22°C, and $V_1$ = 3.17 L. Determine:

a. the net work per cycle,
b. the power developed by the engine if the cycle repeats 1000 times per minute,
c. and the thermal efficiency of the cycle.

SOLUTION:




First, determine the mass of air in the cylinder using the ideal gas law,

$$m = \frac{p_1 V_1}{R T_1}, \quad (1)$$

Using the given values with $R$ = 0.287 kJ/(kg.K),

$m = 3.5570 \times 10^{-3}$ kg.

Now determine the properties at each state:

State 1:

$p_1$ = 95 kPa (abs), $T_1$ = 22°C = 295 K, and $V_1$ = 3.17 L

=> $u_1$ = 210.5 kJ/kg and $v_r(T_1 = 295\text{ K})$ = 647.9 (from the Ideal Gas Table (IGT) for air)

State 2:

$V_2 = V_1 - 3.0$ L = 0.17 L (given that the displacement volume is 3 L), (2)

$$\frac{v_2}{v_1} = \frac{V_2}{V_1} = \frac{v_r(T_2)}{v_r(T_1)} \Rightarrow v_r(T_2) = v_r(T_1)\left(\frac{V_2}{V_1}\right), \quad (3)$$

where $V_1$ = 3.17 L, $V_2$ = 0.17 L,

⇨ $v_r(T_2)$ = 34.745 => $T_2$ = 896.15 K, $u_2$ = 671.405 kJ/kg, $h_2$ = 928.59 kJ/kg (interpolating in the IGT)

The pressure may be found using the ideal gas law,

⇨ $p_2 = \frac{mRT_2}{V_2}$ => $p_2$ = 5381.37 kPa. (4)

State 3:

The cut-off ratio is given as $r_c = 2.5 = V_3/V_2 = T_3/T_2$ => $T_3$ = 2240.4 K, $V_3$ = 0.425 L, (5)

=> $h_3$ = 2553.87 kJ/kg, $u_3$ = 1911.76 kJ/kg, $v_r(T_3)$ = 1.8925 (interpolating in the IGT)

State 4:

$$\frac{v_4}{v_3} = \frac{V_4}{V_3} = \frac{v_r(T_4)}{v_r(T_3)} \Rightarrow v_r(T_4) = v_r(T_3)\left(\frac{V_4}{V_3}\right) = v_r(T_3)\left(\frac{V_4}{V_1}\cdot\frac{V_1}{V_2}\cdot\frac{V_2}{V_3}\right), \quad (6)$$

where $V_4 = V_1$, $V_1$ = 3.17 L (given), $V_2$ = 0.17 L (Eq. (2)), and $V_2/V_3 = 1/r_c = 1/2.5$ (Eq. (5)),

=> $v_r(T_4)$ = 14.1157 => $T_4$ = 1209.8 K and $u_4$ = 942.17 kJ/kg (interpolating in the IGT)

The work into the air during the compression stroke is found by applying the 1st Law to the air (assuming negligible changes in KE and PE and an adiabatic process),

$$m(u_2 - u_1) = W_{in,12} \tag{7}$$

Using the previously calculated values,

$$W_{in,12} = 1.6394 \text{ kJ}.$$

Now calculate the work done by the air during the heat addition and power strokes using the 1st Law,

$$W_{out,23} = p_2(V_3 - V_2), \tag{8}$$

$$m(u_4 - u_3) = -W_{out,34} \tag{9}$$

Using the previously calculated values,

$$W_{out,23} = 1.3722 \text{ kJ and } W_{out,34} = 3.449 \text{ kJ}$$

The net work out is,

$$W_{out,net} = W_{out,23} + W_{out,34} - W_{in,12}, \tag{10}$$

$$\boxed{W_{out,net} = 3.18 \text{ kJ (This is the work over one cycle.)}}$$

Alternately, we could apply the 1st Law over the whole cycle, keeping in mind that the total energy does not change over the cycle,

$$0 = Q_{in,23} - Q_{out,41} + W_{in,12} - W_{out,23} - W_{out,34}, \tag{11}$$

$$0 = Q_{in,23} - Q_{out,41} - W_{out,net}, \tag{12}$$

$$W_{out,net} = Q_{in,23} - Q_{out,41}. \tag{13}$$

The heat transfer into the system during the combustion process is,

$$m(u_3 - u_2) = Q_{in,23} - p_2(V_3 - V_2), \text{ (noting that } p_3 = p_2), \tag{14}$$

$$Q_{in,23} = m(u_3 - u_2) + p_2(V_3 - V_2) = m(h_3 - h_2). \tag{15}$$

Using the previously calculated values,

$$Q_{in,23} = 5.7811 \text{ kJ}.$$

The heat transfer out of the system is,

$$m(u_4 - u_1) = -Q_{out,41}. \tag{16}$$

$$Q_{out,41} = 2.6025 \text{ kJ}.$$

Using the calculated heat values and Eq. (13),

$W_{out,net} = 3.18$ kJ, which is the same value found previously.

The power is,

$$\dot{W}_{out,net} = \left(\frac{W_{out,net}}{1 \text{ cycle}}\right)\left(\frac{1000 \text{ cycle}}{1 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right), \tag{17}$$

$$\boxed{\dot{W}_{out,net} = 53.0 \text{ kJ/s} = 53.0 \text{ kW}}.$$

The thermal efficiency is,

$$\eta = \frac{W_{out,net}}{Q_{in}}, \tag{18}$$

Using $W_{out,net} = 3.18$ kJ and $Q_{in} = 5.7811$ kJ,

⇨ $\boxed{\eta = 0.550 = 55.0\%}$.