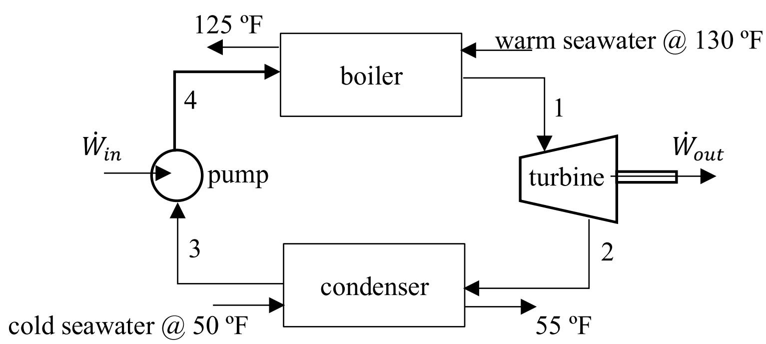
On the island of Hawaii lava flows continuously into the ocean. It is proposed to anchor a floating power plant offshore of the lava flow that uses ammonia as the working fluid. The plant would exploit the temperature variation between the warm water near the surface at 130 ºF and seawater at 50 ºF from a depth of 500 ft to produce power. Using the properties of pure water for the seawater and modeling the power plant as a Rankine cycle, determine:

a. the plant's thermal efficiency, and
b. the mass flow rate of ammonia in lb_m/min, for a net power output of 300 hp.
c. the mass flow rates of seawater through the boiler and condenser, in lb_m/min.

For a related story, see:
https://www.scientificamerican.com/article/hawaii-first-to-harness-deep-ocean-temperatures-for-power/



Working fluid: ammonia

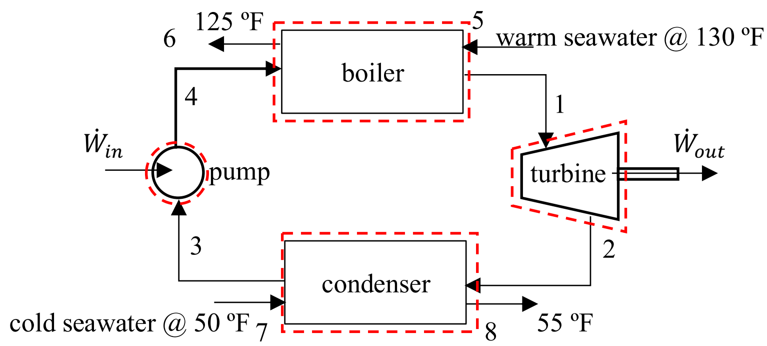
State 1:
T_1 = 120 ºF
saturated vapor

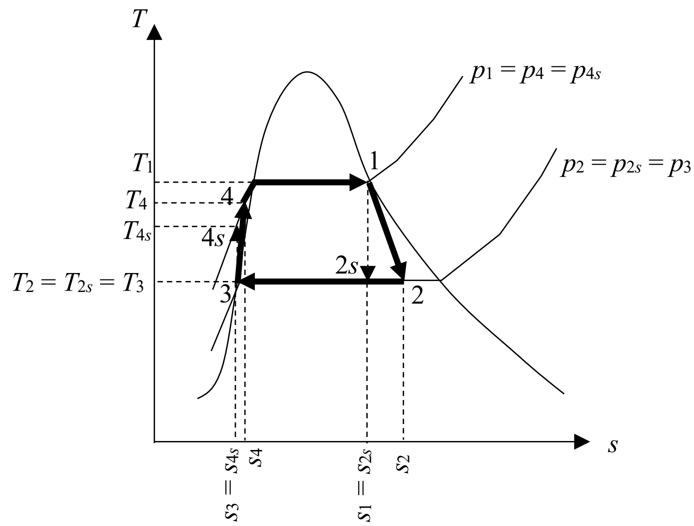
State 2:
T_2 = 60 ºF

State 3:
$p_3 = p_2$
saturated liquid

isentropic turbine efficiency = 0.80
isentropic pump efficiency = 0.85

SOLUTION:


125 ºF
6
5
warm seawater @ 130 ºF
boiler
4
1
$\dot{W}_{in}$
pump
turbine
$\dot{W}_{out}$
3
2
condenser
cold seawater @ 50 ºF
7
8
55 ºF


T
$p_1 = p_4 = p_{4s}$
T_1
1
$p_2 = p_{2s} = p_3$
T_4
4
T_{4s}
4s
2s
$T_2 = T_{2s} = T_3$
3
2
s
$s_3 = s_{4s}$
s_4
$s_1 = s_{2s}$
s_2

First find the temperatures, specific enthalpies, and specific entropies at each of the states using the property tables for ammonia.

State	***p*** **[psia]**	***T*** **[°F]**	**Phase**	***x*** **[-]**	***h*** **[Btu/lb$_m$]**	***s*** **[Btu/(lb$_m$. °R)]**
1	286.47 (= p_{sat})	120	sat. vapor	1	632.95	1.1405
2	107.66 (= p_3)	60	SLVM°,=	0.932429	591.626	1.16038
2*s*	107.66 (= p_2)	60	SLVM°	0.912476	581.295	1.1405 (= s_1)
3	107.66 (= p_{sat})	60	sat. liquid	0	108.87	0.2314
4	286.47 (= p_1)	61.08	CL+	N/A	109.881	0.23374
4*s*	286.47 (= p_4)	60	CL+	N/A	109.729	0.2314 (= s_3)

°For a SLVM,

$$x = \frac{s - s_f}{s_g - s_f}, \quad (1)$$

$$h = (1 - x)h_f + xh_g. \quad (2)$$

State 2*s*:

T_{2s} = 60 °F, $s_{2s} = s_1$ = 1.1405 Btu/(lb$_m$.°R)]; s_{f2s} = 0.2314 Btu/(lb$_m$.°R), s_{g2s} = 1.2277 Btu/(lb$_m$.°R) => x_{2s} = 0.912476.

h_{f2s} = 108.87 Btu/lb$_m$, h_{g2s} = 626.61 Btu/lb$_m$ => h_{2s} = 581.295 Btu/lb$_m$.

State 2:

T_2 = 60 °F, h_2 = 591.626 Btu/lb$_m$ (see below); h_{f2} = 108.87 Btu/lb$_m$, h_{g2} = 626.61 Btu/lb$_m$ => x_2 = 0.932429.

s_{f2} = 0.2314 Btu/(lb$_m$.°R), s_{g2} = 1.2277 Btu/(lb$_m$.°R) => s_2 = 1.16038 Btu/(lb$_m$.°R).

=To find the conditions at State 2, make use of the turbine isentropic efficiency,

$$\eta_{turb.,isen.} = \frac{\dot{W}_{out}}{\dot{W}_{out,isen}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{turb.,isen.}(h_1 - h_{2s}) = 591.626 \text{ Btu/lb}_m, \quad (3)$$

where h_1 = 632.95 Btu/lb$_m$, h_{2s} = 581.295 Btu/lb$_m$, and $\eta_{turb.,isen.}$= 0.80.

+ For a compressed liquid,

$$h_{CL}(T,p) \approx h_f(T) + [p - p_{sat}(T)]v_f(T) \quad \text{and} \quad s_{CL}(T,p) \approx s_f(T) \quad (4)$$

State 4*s*:

p_{4s} = 286.47 psia, $s_{4s} = s_3$ = 0.2314 Btu/(lb$_m$.°R) =>

T_{4s} = 60 °F, $p_{sat,4s}$ = 107.66 psia, v_{f4s} = 0.02597 ft^3/lb$_m$, h_{f4s} = 108.87 Btu/lb$_m$ => h_{4s} = 109.729 Btu/lb$_m$.

=To find the conditions at State 4, make use of the pump isentropic efficiency,

$$\eta_{pump,isen.} = \frac{\dot{W}_{in,isen}}{\dot{W}_{in}} = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{h_{4s} - h_3}{\eta_{pump,isen.}} = 109.881 \text{ Btu/lb}_m, \quad (5)$$

where h_3 = 108.87 Btu/lb$_m$, h_{4s} = 109.729 Btu/lb$_m$, and $\eta_{pump,isen.}$= 0.85. The temperature corresponding to this specific enthalpy is, after some linear interpolation, T_4 = 61.08 °F, and the specific entropy is, s_4 = 0.23374 Btu/(lb$_m$.°R).

Now apply the 1st Law to a control volume surrounding the turbine,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out}, \quad (6)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (7)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_1 - h_2), \quad (8)$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_1 = \dot{m}_2 = \dot{m}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \quad (9)$$

$$\dot{W}_{out} = ?. \quad (10)$$

Substitute and solve for the power,

$$\frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2. \quad (11)$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{W}_{out}}{\dot{m}} = 41.324 \text{ Btu/lb}_m.$$

Apply the 1st Law to a control volume surrounding the pump,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} + \dot{W}_{in}, \quad (12)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (13)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_3 - h_4), \quad (14)$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_3 = \dot{m}_4 = \dot{m}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \quad (15)$$

$$\dot{W}_{in} = ?. \quad (16)$$

Substitute and solve for the power,

$$\frac{\dot{W}_{in}}{\dot{m}} = h_4 - h_3. \quad (17)$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{W}_{in}}{\dot{m}} = 1.011 \text{ Btu/lb}_m.$$

Apply the 1st Law to a CV surrounding the boiler,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out}, \quad (18)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (19)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_4 - h_1), \quad (20)$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_1 = \dot{m}_4 = \dot{m}$)

$$\dot{Q}_{in} = ?, \quad (21)$$

$$\dot{W}_{out} = 0 \quad \text{(the steam generator is a passive device)}. \quad (22)$$

Substitute and solve for the rate of heat transfer,

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4. \quad (23)$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{Q}_{in}}{\dot{m}} = 523.069 \text{ Btu/lb}_m.$$

Using the power in and power out results,

$$\frac{\dot{W}_{out,net}}{\dot{m}} = \frac{\dot{W}_{out}}{\dot{m}} - \frac{\dot{W}_{in}}{\dot{m}} = 40.313 \text{ Btu/lb}_m \quad (24)$$

The thermal efficiency for the power cycle is,

$$\boxed{\eta = \frac{\dot{W}_{out,net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.0771 = 7.71\%} \quad (25)$$

This thermal efficiency is less than the Carnot cycle thermal efficiency of

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 0.136 = 13.6\%, \quad (26)$$

where T_C = 509.67 ºR (= 50 ºF) and T_H = 589.67 ºR (= 130 ºF). The Rankine cycle efficiency is smaller than the Carnot cycle efficiency because of irreversibilities in the cycle. Even if the Rankine cycle was ideal (isentropic conditions across the pump and turbine), it would still have a smaller efficiency than the Carnot cycle because the average temperature during heat addition is smaller than that for a Carnot cycle, i.e., the average temperature from State 4 to State 1 is smaller than the average temperature from State 4 to State 1 in a Carnot cycle.

The mass flow rate in the cycle can be determined using Eq. (24) and the given net power output of $\dot{W}_{out,net} = 300$ hp,

$$\frac{\dot{W}_{out,net}}{\dot{m}} = 40.313 \text{ Btu/lb}_m \Rightarrow \boxed{\dot{m} = \frac{300 \text{ hp}}{40.313 \text{ Btu/lbm}} = 316 \text{ lb}_m\text{/min}}. \quad (27)$$

Now determine the mass flow rate of the cooling water for the boiler. Apply a control volume around the boiler and apply the 1st Law,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} + \dot{W}_{in}, \quad (28)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (29)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_4 - h_1) + \dot{m}_{b,sw}(h_5 - h_6), \quad (30)$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_4 = \dot{m}_1 = \dot{m}$ and $\dot{m}_5 = \dot{m}_6 = \dot{m}_{b,sw}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \quad (31)$$

$$\dot{W}_{in} = 0 \quad \text{(the device is passive)}. \quad (32)$$

Substitute and solve for the boiler seawater mass flow rate,

$$0 = \dot{m}(h_4 - h_1) + \dot{m}_{b,sw}(h_5 - h_6), \quad (33)$$

$$\dot{m}_{b,sw} = \dot{m}\left(\frac{h_4 - h_1}{h_6 - h_5}\right). \quad (34)$$

The mass flow rate for the cycle was found in Eq. (27). The specific enthalpies for States 4 and 1 are given in the table at the start of this solution. Not enough information is given to determine the specific enthalpies for States 5 and 6 using the compressed liquid approximation; however, since the temperature is small and seawater can be reasonably assumed to be incompressible, let,

$$\Delta h = \Delta u + v\Delta p = \Delta u = c\Delta T. \quad (35)$$

where $\Delta p = 0$ since the pressure of the surrounding seawater at the inlet and outlet to the boiler is approximately the same. The specific heat for seawater is found from a property table to be c = 0.999 Btu/(lb$_m$.ºR). Using T_5 = 130 ºF and T_6 = 125 ºF along with the previously determined values,

$$\boxed{\dot{m}_{b,sw} = 33000 \text{ lb}_m\text{/min}}.$$

Performing a similar 1st Law analysis for a CV surrounding the condenser, but with c = 1.005 Btu/(lb$_m$.ºR),

$$\dot{m}_{c,sw} = \dot{m}\left(\frac{h_2 - h_3}{h_8 - h_7}\right), \quad (36)$$

$$\boxed{\dot{m}_{c,sw} = 30300 \text{ lb}_m\text{/min}}.$$

The efficiency of this power plant is small (7.71%). Even for an ideal Carnot cycle the efficiency is small (13.6%). The small cycle thermal efficiency coupled with the large mass flow rates required for pumping the seawater (decreasing the net power out of the cycle even further) and the material and structural costs for operating in corrosive seawater make for a weak incentive to construct and operate this powerplant from a financial point of view.